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Agglomeration, city size and crime ☆

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ABSTRACT

This paper analyzes the relationship between crime and agglomeration where the land, labor, product, and crime markets are endogenously determined. Our main theoretical findings are the following: (i) better accessibility to jobs decreases crime in the short run but may increase crime in the long run; (ii) the per-capita crime rate increases with city size; (iii) when allowing for endogenous policing, lower commuting costs make the impact of police on crime more efficient.

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1. Introduction

Assuming that individuals are rational decision-makers who engage in either legal or illegal activities according to the expected utility from each activity, the economic literature shows that different crime-fighting urban policies can be implemented. First, more police resource may reduce the crime rate by reducing the net benefits of crime because of a higher risk of detection and punishment. Some empirical studies reveal that an increase in the urban police force produced a 3% to 10% long-term decline in crime rates (Levitt, 1997).[1] Second, better access to legal labor markets may raise the opportunity cost of illegal activity. Third, institutions may influence the aversion to illegal activities of individuals.

However, the analysis of the crime-reduction strategy fails to take into account the effects of space (*i.e.* the location of jobs and people) on criminal activities. Crime is an important social problem but also an urban phenomenon. It is well documented that there is *relatively* more crime in big than in small cities (Glaeser and Sacerdote, 1999; Kahn, 2010).[2] For example, the rate of violent crime in cities with more than 250,000 inhabitants is 346 per 100,000 inhabitants whereas in

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[1] Instrumenting with election cycles to take account of the endogeneity of police staffing, Levitt (1997) finds that the elasticity of violent crime with respect to sworn officers is estimated to be approximately −1.0 while, for property crime, the elasticity is around −0.3. See also the natural experiment of Di Tella and Schargrodsky (2004) who look at a redeployment in Buenos Aires that followed the bombing of a Jewish center in 1994. Car thefts fell by 75 percent on the blocks where the extra police were stationed, and did not rise elsewhere. Klick and Tabarrok (2005) for Washington, D.C., Poutvaara and Priks (2009) for Stockholm, and Draca et al. (2011) for London find similar results.

[2] Glaeser and Sacerdote (1999) use police jurisdictions as their geographic units while Kahn (2010) uses counties.

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cities with less than 10,000 inhabitants, the rate of violent crime is just 176 per 100,000 (Glaeser, 1998). Similar figures can be found for property crimes or other less violent crimes. Agglomeration creates a multiplier effect on the crime rate through two channels. On the one hand, as mentioned in Glaeser and Sacerdote (1999), a larger city size induces greater expected pecuniary returns because criminals face a larger number of potential victims and a lower probability of being arrested. On the other hand, more workers in a city increase the land rents or the commuting costs, thereby diminishing the opportunity cost of illegal activity.

The aim of this paper is to propose a model that captures some of the stylized facts observed in real-world cities and to analyze policies aiming at reducing crime in a spatial context. Our model delivers a full analytical solution that captures in a simple way how interactions among land, product, crime and labor markets yield agglomeration and criminal activity.

For this purpose, we develop a model where city size and the type of activities (crime and labor) are endogenous within a full-fledged general equilibrium model. Individuals are freely mobile between and within the cities. We consider four different markets in each city: land, labor, product, and crime. The land market is assumed to be competitive and land is allocated to the highest bidders in each city. Land is owned by absentee landlords. The labor market is also competitive and wages are determined by free entry. Monopolistic competition prevails in the product market, which implies that each firm has a monopoly power on her variety. Finally, the crime market is competitive and the mass of criminals is determined by a cost–benefit analysis for each person. Hence, a land market, spatial frictions, and agglomeration economies are introduced in our general equilibrium model.

Let us be more precise. We first develop a framework where the city size measured as population is exogenous (Section 3). Although, under this configuration, our model has a partial equilibrium flavor, it is rich enough to take into account the following fundamental aspects of urban development: larger cities are associated with higher nominal wages (Baum-Snow and Pavan, 2011), more varieties (Handbury and Weinstein, 2015), higher housing and commuting costs (Fujita and Thisse, 2013) and a higher crime rate (Glaeser and Sacerdote, 1999). Individuals are heterogeneous in their incentives to commit crime. They freely choose their location within the city and decide whether to become a criminal or a worker. In Section 4, we show the following results: (i) higher commuting costs or, equivalently, worse job access lead to more criminal activities in the city; (ii) the impact of commuting costs on criminal activities is higher when the city size increases; and (iii) criminal activities increase more than proportionally when there is an increase in city size.

Most of these results are empirically documented. For example, concerning (i), using 206 census tracts in the city of Atlanta and Dekalb county and a state-of-the-art job accessibility measure, Ihlanfeldt (2002, 2006, 2007) demonstrates that modest improvements in the job accessibility of male youth, in particular blacks, cause marked reductions in crime, especially within the category of drug-abuse violations. He found an elasticity of 0.361, which implies that 20 additional jobs will decrease the neighborhood's density of drug crime by 3.61%. If we now consider (iii), then this is true in most cities in the world. Glaeser and Sacerdote have shown that this was true for the United States. Similarly, looking at U.S. metropolitan areas, O'Flaherty and Sethi (2015) show (see their Table 9) that this is true for motor vehicle theft and robbery and their elasticities were 0.23 and 0.33, respectively, both significant, and not far from the Glaeser–Sacerdote elasticity. If we now look at European cities, the same pattern emerges. Fig. 1a and b documents this pattern for France for offences against persons (Fig. 1a) and property crimes (Fig. 1b) and where the spatial unit is the department.

Nevertheless, we have to discuss the robustness of the results by considering that households have two options in response to crime risk: they can vote for anti-crime policies or they can vote with their feet (Linden and Rockoff, 2008). In Section 5, we introduce an urban government that levies a tax on workers in order to finance policy resources to fight criminal activities. We show that there is a U-shaped relationship between the crime rate in the economy and the tax rate so that increasing resources to fight crime financed by taxes on workers can backfire if the tax rate is too high. We also show that *lower commuting costs or better job access make the impact of police on crime more efficient*.

These are short-run effects. In the long run, the individuals can migrate to avoid the high levels of crime in the large city. In Section 6, we extend our model by considering a system of two cities where the city size is endogenous because of the mobility of individuals between the two cities. This extension implies that even if cities are *ex ante* identical, the *ex post* differences in the crime rate, the economic structure and the population size across cities emerge as the unintentional outcome of a myriad of decisions made by firms and households pursuing their own interest. We characterize the three different stable equilibria as a function of commuting costs. When the commuting costs are high enough, the population of workers and criminals is evenly distributed between the two cities. When the commuting costs take intermediate values, there is a large and a small city where there are more criminals and workers in the former than in the latter. When the commuting costs are low enough, there is a single city. This means that when the commuting costs take intermediate values, even though the level of criminal activity creates a negative externality on workers, agglomeration takes place and attracts the majority of workers and the largest city has the higher share of criminals.

This framework allows us to study the effects of lower commuting costs in each city on criminal activity in the economy. If a decrease in commuting costs (or better access) reduces crime in the short run (when the city size is unchanged) because the urban costs experienced by workers decline, this is no longer true in the long run when agents are perfectly mobile between cities. Indeed, a reduction in commuting costs induces more people and jobs to move to the larger city. In that case, a decrease in commuting costs has an ambiguous effect on crime since, in bigger cities, people earn higher wages but also experience higher urban costs and obtain higher proceeds from crime. We show that criminal activity increases in the economy when commuting costs decline only if the size of the agglomeration is high enough. This implies that the improvement of transportation will decrease crime only in very large cities where agglomeration is important but the global

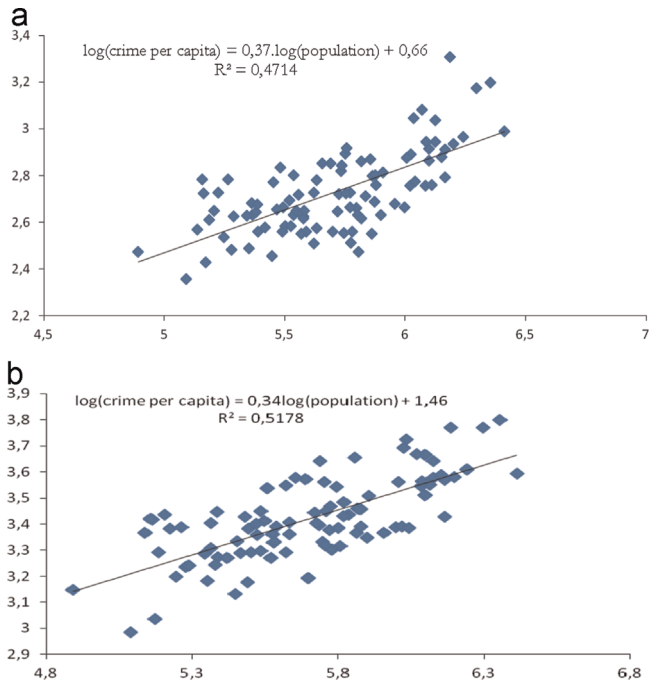

Fig. 1. (a) Offences against persons per 1000 inhabitants and population size in France. *Source*: Direction Centrale de la Police Judiciaire, 2008 and INSEE. Authors' calculations. *Note*: Offences against persons include homicide, voluntary or involuntary battery and injuries, offences against public decency (including pimping, rape, sexual assault), offences against families and children (including violence, illtreatment, abandonment), as well as the taking of hostages, unlawful confinement, abduction, threats and blackmail, offences against dignity and personality, etc. The spatial unit is Département (French administrative area; continental France is partitioned into 94 Départements). (b) Property crimes per 1000 inhabitants and population size in France. *Source*: Direction Centrale de la Police Judiciaire, 2008 and INSEE. Authors' calculations. *Note*: Property crimes involve theft of property such as burglary, larceny, auto theft, and arson. The spatial unit is Département (French administrative area; continental France is partitioned into 94 Départements).

effect is in general ambiguous. As noted above, Ihlanfeldt (2002, 2006, 2007) shows that for the city of Atlanta in the United States, an improvement in job access (lower commuting costs) always decreases crime. Studying the city of Bogotá, Columbia Olarte Bacares (2014) shows that this effect is not always positive and depends on the type of crime committed. Indeed, in 2000, Bogotá implemented an ingenious and innovative Bus Rapid Transport (BRT) system called Transmilenio (TM) but this only affected some areas of the city. Olarte Bacares (2014) shows that the presence of the Transmilenio system in an area *increases* the number of "thefts from people" as compared to areas without a TM system while it *decreases* house breaking. This means that policy makers should be careful in implementing a transportation policy because it may backfire on crime. Our paper provides an explanation for why this might be the case.

2. Related literature

To our knowledge, three types of theoretical models have integrated space and location in criminal behavior. First, *social interaction* models, which state that individual criminal behavior does not only depend on individual incentives but also on the behavior of peers and neighbors, are a natural way of explaining the concentration of crime by area. An individual is more likely to commit crime if her peers commit crime than if they do not (Glaeser et al., 1996; Calvó-Armengol and Zenou, 2004; Ballester et al., 2006, 2010; Calvó-Armengol et al., 2007; Patacchini and Zenou, 2012). This explanation is backed up by several empirical studies showing that neighbors do indeed matter in explaining crime behaviors. Using the 1989 NBER survey of young living in low-income, inner-city Boston neighborhoods, Case and Katz (1991) found that residence in a neighborhood where many other youths are involved in crime is associated with an increase in an individual's probability of committing crime. Exploiting a natural experiment (*i.e.* the Moving to Opportunity experiment that has assigned a total of 614 families living in high-poverty Baltimore neighborhoods into richer neighborhoods), Ludwig et al. (2001) and Kling et al. (2005) find that this policy reduces juvenile arrests for violent offences by 30–50 percent, relative to a control group. This also suggests very strong social interactions in crime behaviors. Bayer et al. (2009) consider the influence of juvenile offenders serving time in the same correctional facility on each other's subsequent criminal behavior. They also find strong evidence of learning effects in criminal activities since exposure to peers with a history of committing a particular crime increases the probability that an individual who has already committed the same type of crime recidivates that crime.

Patacchini and Zenou (2012) find that peer effects in crime are strong, especially for petty crimes.[3] More recently, Damm and Dustmann (2014) investigate the following question: Does growing up in a neighborhood where a relatively large share of youth has committed crime increase the individual's probability of committing crime later on? To answer this question, Damm and Dustmann exploit a Danish natural experiment that randomly allocates parents of young children to neighborhoods with different shares of youth criminals. They find that a one standard deviation increase in the share of youth criminals in the municipality of initial assignment increases the probability of male refugees being charged with an offense between ages 15 and 21 by 5–9 percent and the number of convictions by 7–11 percent.[4]

Second, Freeman et al. (1996) provide a theoretical model that explains why criminals are concentrated in some areas of the city (ghettos) and why they tend to commit crimes in their own local areas and not in rich neighborhoods. Their explanation is based on the fact that when criminals are numerous in an area, the probability of being caught is low so that criminals create a positive externality for each other. In this context, *criminals concentrate their effort in (poor) neighborhoods where the probability of being caught is small*.[5] This explanation also has strong empirical support (see *e.g.* O'Sullivan, 2000).

Finally, Verdier and Zenou (2004) show that prejudices and *distance to jobs* (legal activities) can explain crime activities, especially among blacks. If everybody believes that blacks are more criminal than whites – even if there is no basis for this – then blacks are offered lower wages and, as a result, locate further away from jobs. Distant residence increases the black–white wage gap even more because of more tiredness and higher commuting costs. Blacks thus have a lower opportunity cost of committing crime and indeed become more criminal than whites. In this model, beliefs are self-fulfilling.[6]

Our contribution is different since we focus on the impact of inter-city mobility, city size and agglomeration effects on criminal behavior. We believe this to be the first model that integrates crime and agglomeration economics in a unified framework by modeling the labor, crime, land and product market. In particular, our model enables us to (i) reproduce different stylized facts observed in real-world cities by showing that there exist big cities with both a high number of workers and criminals and (ii) study the efficiency of different policies aiming at reducing crime in an urban context. One of our main results is to show that better accessibility to jobs decreases crime in the short run but may increase it in the long run.[7]

Our model is also related to the economic geography literature and can be viewed as a contribution to the theory of agglomeration with heterogeneous agents (for a recent survey, see Behrens and Robert-Nicoud, 2015). Indeed, while much attention has been devoted to the impact of commuting costs and pollution on the spatial organization of the economy, rather little consideration has been given to the interactions between crime and urbanization. Yet, such crime as pollution and congestion is an important factor that discourages the growth of cities (Tolley, 1974; Kahn, 2010).[8]

3. The model

Our model describes a city where each individual chooses where to locate and whether to become a criminal or not. Each individual can either be a worker or a criminal but not both. We consider four different markets: land, labor, product, and crime. As is standard in the urban literature (Zenou, 2009; Fujita and Thisse, 2013), the land market is assumed to be competitive and land is allocated to the highest bidders in the city. Land is owned by absentee landlords. The labor market is also competitive and wages are determined by free entry. Monopolistic competition prevails in the product market, which implies that each firm has a monopoly power on her variety. Finally, the crime market is competitive and the mass of criminals is determined by a cost–benefit analysis exerted by each individual.

The city is formally described by a one-dimensional space. It can accommodate firms, criminals and workers. The masses of criminals and workers are denoted by C and L, respectively. The city has a Central Business District (CBD) located at $x=0$ where all firms are established.[9] Individuals live between the CBD and the city fringe. Without loss of generality, we focus on the right-hand side of the city, the left-hand side being perfectly symmetric. Distances and locations are expressed by the same variable x measured from the CBD.

For simplicity, we assume that each individual consumes a residential plot of fixed size (normalized to 1), regardless of her location and her status (criminal or worker). This assumption is often adopted in frameworks where the land market is explicitly modeled because it captures the basic trade-off between long/short commutes and low/high land rents. Notice

[3] See also Helsley and Zenou (2014) and Sato and Zenou (2015) who show that the interaction between social and geographical space can " amplify" the behavior of individuals.

[4] See also the very interesting paper by Corno (2015) which studies peer effects in crime among the homeless in Milan. To instrument for peer effects, she uses rainfall since it fosters concentration of homeless individuals in sheltered places and increases the probability of meetings. Corno finds that the probability of arrest decreases by 16 percentage points with network size.

[5] See also Deutsch et al. (1987).

[6] For more detailed surveys on the spatial aspects of crime, see Zenou (2003), Raphael and Sills (2005) and O'Flaherty and Sethi (2015).

[7] Using a different model, O'Sullivan (2005) shows that gentrification results from a decrease in crime or an increase in the frequency of travel to the center.

[8] Observe that contrary to the cross-city quality-of-life literature (see, for example, Desmet and Rossi-Hansberg, 2013), we here endogenize the "production" of city amenities.

[9] See the survey by Duranton and Puga (2004) for the reasons explaining the existence of a CBD.

that our main results would not change if land consumption were endogenous. Let $\lambda = C + L$ denote the population residing in the city so that the size of the right-hand side of this city is given by $\lambda/2$.

3.1. Preferences and budget constraints

Individuals are heterogeneous in their incentives to commit crime. They have different aversions to crime, denoted by c, so that a higher c means more aversion towards crime. We assume that c is uniformly distributed on the interval [0,1].[10] The parameter c could be interpreted in numerous other ways (the ability to commit crime, the disutility of working, etc.).

Individuals consume two types of goods: a *homogeneous* good and *non-tradeable* goods (where trade costs are prohibitive), which are horizontally differentiated by varieties.[11] One can think of a bundle of locally produced *services* such as, for example, restaurants, retail shops, and theaters (Glaeser et al., 2001). The preferences are the same across individuals and, for $v \in [0, n]$, the utility of a consumer is given by

$$U(q_0; q(v)) = \alpha \int_0^n q(v)\,\mathrm{d}v - \frac{\gamma}{2n}\left(\int_0^n q(v)\,\mathrm{d}v\right)^2 - \frac{(\beta-\gamma)}{2}\int_0^n [q(v)]^2\,\mathrm{d}v + q_0 \tag{1}$$

where $q(v)$ is the quantity of variety v of services and q_0 the quantity of the homogeneous good, which is taken as the *numéraire*. All parameters α, β and γ are positive; $\gamma > 0$ measures the substitutability between varieties, whereas $\beta - \gamma > 0$ expresses the intensity of the love for variety. Eq. (1) is a quasi-linear utility function with a quadratic sub-utility, symmetric in all varieties, used in Tabuchi and Thisse (2006) and Gaigné and Thisse (2009), which slightly differs from that used by Ottaviano et al. (2002). Although this modeling strategy gives our framework a partial equilibrium flavor, it does not remove the interaction among product, labor, land and crime markets, thus allowing us to develop a full-fledged model of agglomeration formation, independent of the relative size of the service sector.[12] Note that the utility (1) degenerates into a utility function that is quadratic in total consumption $\int_0^n q(v)\,\mathrm{d}v$ when $\beta = \gamma$.

Each worker commutes to the CBD and pays a unit commuting cost per unit of distance of $t > 0$, so that a worker located at $x > 0$ bears a commuting cost equal to tx. The budget constraint of a *worker* residing at x is then given by[13]

$$\int_0^n p(v)q(v)\,\mathrm{d}v + q_0 + R^w(x) + tx = I - \xi C + \overline{q}_0 \tag{2}$$

where $p(v)$ is the price of the service good for variety v, $R^w(x)$ is the land rent paid by workers (superscript w) located at x and I is the income of a worker. The homogeneous good is available as an endowment denoted by $\overline{q}_0$. In this formulation, C is the mass of criminals while ξ is a lump-sum amount stolen by each criminal. In other words, we assume that there are *negative externalities* of having criminals around so that the higher is the number of criminals, the higher are these negative externalities. On average, the stolen amount per worker increases with the mass of criminals in the city. In this formulation, each worker is "visited" by C criminals who each takes ξ. By the law of large numbers, this means that, on average, a worker meets C criminals. This also implies that each criminal "robs" L workers and takes ξ from each worker so that her average proceeds from crime are ξL.

Each worker chooses her location so as to maximize her utility (1) under the budget constraint (2).

The budget constraint of a *criminal* residing at x is given by

$$\int_0^n p(v)q(v)\,\mathrm{d}v + q_0 + R^c(x) = \xi L + \overline{q}_0 \tag{3}$$

where L is the mass of workers and $R^c(x)$ is the land rent paid by criminals (superscript c) located at x. Observe that individuals are specialized here so that workers only work and do not commit crime while criminals only commit crime. As mentioned above, from Eq. (3), we see that the proceeds from crime ξL are increasing in the number of workers L in the city. For simplicity and to be consistent with (2), each criminal is assumed to steal a fraction ξ from these workers. Hence, there is no *direct* competition between criminals. However, as we will show below, they are indirectly connected via the mass of workers in a city. Indeed, more criminals induce less workers and, in turn, lower pecuniary returns per criminal. In other words, the income of each criminal depends on the mass of criminals in each city.

The assumption that criminals do not bear a commuting cost is just made for simplicity. This assumption could be viewed as inconsistent with the assumption that each criminal steals a given amount ξ from each worker, which implies that criminal activity should involve commuting. One easy way of justifying this assumption is to assume that, from time to time,

[10] See Verdier and Zenou (2004) and Conley and Wang (2006) for models with heterogeneous disutilities of crime.

[11] We could consider a model where the differentiated product is traded among cities. However, such a model would make the formal analysis more complex while the gains in terms of results would be very limited. Indeed, as shown in Gaigné and Thisse (2014), even in the absence of trade, agglomeration and dispersion forces are similar.

[12] This structure of preferences is not essential to many of the results of this paper. We could use a traditional Dixit–Stiglitz structure. However, the quasi-linear type of preference is superior to the CES approach when the city size is considered as endogenous (Section 6). Indeed, the approach adopted in this paper allows us to deliver analytical solutions that capture the relationships among inter-city migration, city size and criminal activity in a simple way.

[13] Note that housing and commuting costs account for a large share of consumer expenditures. Housing on average accounts for 26% of household budgets in the United States (23% in France) while 18% of total expenditures (15% in France) are spent on car purchases, gasoline, and other related expenses which do not include the cost of time spent traveling (Bureau of Labor Statistics, 2012).

workers go to the "criminal area" (the city boundary between $L/2$ and $N/2$) and are robbed there. A more convincing way is to assume that all crimes occurred in the CBD. Then, since all criminals reside in the same area of the city, we could assume that, within this area, the commuting costs are constant and equal to Θ so that each criminal has to pay a commuting cost equal to $\Theta + t'L/2$ to commute to the CBD and commit her crimes there, where t' is the commuting cost per unit of distance for each criminal. This term would have to be added to the left-hand side of Eq. (3). Within this framework, we can show that all our results hold if t' (the criminal's commuting cost) is lower than t (the worker's commuting cost), which can be justified by the fact that criminals indeed use cheaper commuting modes or commute less frequently than workers.

3.2. Technology and market structure

Each variety of services v is supplied by a single firm and any firm supplies a single differentiated service under monopolistic competition. Labor is the only production factor. Each firm requires a fixed amount of labor and thus operates under increasing returns. We choose the unit of labor for the fixed requirement to be equal to 1. For simplicity, the marginal requirement is normalized to zero. By implication, the profit made by a service firm v is given by

$$\pi(v) = p(v)q(v)\lambda - I \tag{4}$$

where $p(v)$ is the price quoted by a service firm v and I is the wage paid by a service firm to her workers. Consistent with (2) and (3), formulation (4) means that both criminals and workers consume all goods. As we will see below, workers and criminals consume the same level of each variety because, given our utility function, the demand for differentiated services does not depend on income.

3.3. Services market, equilibrium prices and consumer surplus

The maximization of utility (1) under the budget constraint (2) or (3) leads to the demand for a service v given by

$$q(v) = \frac{\alpha}{\beta} - \frac{p(v)}{\beta - \gamma} + \frac{\gamma}{\beta(\beta-\gamma)}\frac{\mathbf{P}}{n} \tag{5}$$

where the price index $\mathbf{P} = \int_0^n p(v)\,\mathrm{d}v$ is defined over the range of services produced in the city because this good is non-tradeable. Since the demand for each differentiated product does not depend on the net income (wage minus land rent and commuting costs) of each individual, it does not matter if the budget constraint is (2) or (3).

Each service firm determines its price by maximizing (4), using (5) and treating the price index $\mathbf{P}$ as a parameter. Solving the first-order conditions yields the equilibrium prices of a *non-tradeable* service for a variety in the city, given by

$$p^* = \frac{\alpha(\beta-\gamma)}{\beta + (\beta-\gamma)} \tag{6}$$

which is the same for all varieties. Hence, the consumer surplus generated by any variety at the equilibrium market price p^* is equal to

$$S^*(v) = \frac{q^*(v)[p(0) - p^*]}{2} = \frac{(\alpha - p^*)^2}{\beta} = \frac{\alpha^2\beta^2}{[\beta + (\beta-\gamma)]^2 + \beta}$$

where $p(0)$ is the inverse demand when $q(v) = 0$. Note that the consumer surplus $S^*(v)$ for any variety is the same because all varieties are available at the same price. However, the consumer surplus generated by *all* varieties available in the city, *i.e.* nS^*, changes with the supply of varieties in the city. Without loss of generality, we then set $S^* = 1$. This assumption does not qualitatively affect the properties of the spatial equilibria but greatly simplifies the algebra.

3.4. Urban labor market and equilibrium wages

Because labor is the only factor of production, the number of varieties available in each city is proportional to the mass of individuals living *and* working in this city. More precisely, the labor market-clearing conditions imply

$$n = L. \tag{7}$$

Urban labor markets are local and the equilibrium wage is determined by a bidding process where firms compete for workers by offering them higher wages until no firm can profitably enter the market. In other words, the operating profits are completely absorbed by the wage bill. This is a free-entry condition that sets profits (4) equal to zero so that, using (5) and (6), we find that the equilibrium wage paid by service firms established in the city is equal to

$$I^* = \varphi\lambda \tag{8}$$

where

$$\varphi \equiv (p^*)^2 = \left[\frac{\alpha(\beta-\gamma)}{\beta + (\beta-\gamma)}\right]^2. \tag{9}$$

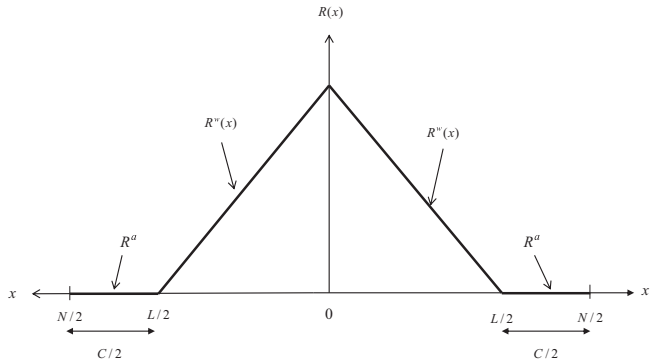

Fig. 2. Equilibrium land use.

In accordance with empirical evidence, the equilibrium wage I^* increases with population size ($\lambda = L + C$). However, the equilibrium wage is unaffected by the residential location of each worker within the city. It is also worth stressing that the equilibrium wage rises with product differentiation (low γ).

3.5. Land market and equilibrium land rents

Let us first determine the equilibrium land rent for *workers*. From the budget constraint (2), we obtain

$$q_0 = I - R^w(x) - tx - \xi C + \overline{q}_0 - \int_0^n p(v)q(v)\,\mathrm{d}v.$$

By plugging this value and the equilibrium quantities and prices (5) and (6) into the utility (1), we obtain

$$V^w(x) = n + I^* - R^w(x) - tx - \xi C + \overline{q}_0. \tag{10}$$

where I^* is defined by (8). Because of the fixed lot size assumption (normalized to 1), the value of the consumption of nonspatial goods $\int_0^n q(v)p(v)\,\mathrm{d}v + q_0$ at the residential equilibrium is the same regardless of the worker's location. Using (2), this implies that the total *urban costs*, $UC^w(x) \equiv R^w(x) + tx$, borne by a worker living at location x in the city, are constant whatever the location x.

Since *criminals* do not commute to the CBD, which implies that their utility does not depend on location x, we have $R^c(x) = R^c$. In equilibrium, since it is costly for workers to be far away from the CBD, they will bid away criminals who will live at the city fringe, paying the opportunity cost of land R^a so that $R^c = R^a$.[14] Without loss of generality, the opportunity cost of land is normalized to zero, *i.e.* $R^a = 0$.

For workers, given $V^w(x)$, the equilibrium land rent in the city must solve $\partial V^w(x)/\partial x = 0$ or, equivalently, $\frac{\partial R^w(x)}{\partial x} + t = 0$, whose solution is $R^w(x) = r_0 - tx$, where r_0 is a constant.[15] Because the opportunity cost of land R^a is equal to zero, it has to be that $R^w(L/2) = 0$ (see Fig. 2) so that $r_0 = tL/2$. As a result, the equilibrium land rent for workers is equal to

$$R^w(x) = t\left(\frac{L}{2} - x\right) \tag{11}$$

and the urban costs borne by a worker are given by

$$UC^w = t\frac{L}{2}. \tag{12}$$

As expected, the urban costs increase with the commuting costs and the mass of workers.

4. Criminal activities and city size

Let $\theta = C/\lambda$ be the share of criminals in the *city*, so that

$$C = \theta\lambda \quad \text{and} \quad L = (1-\theta)\lambda.$$

In this section, θ is endogenously determined for any given population size λ. An individual becomes criminal if and only if $V^c - V^w > 0$, where V^c and V^w are the utility of a criminal and a worker at equilibrium prices, respectively. Plugging the

[14] In our approach, the location of criminals plays no role in the criminal activity. Note that we could consider a polycentric city (so that jobs are also located in subcenters close to the city fringe). Under this configuration, a fraction of criminals will reside between the CBD and the subcenters.

[15] We could easily extend the model to take into account the fact that workers residing further away from criminals experience lower negative externalities. For example, if we assume that $\xi(x) = \xi_0 + \xi_1 x$ so that the criminals steal less from workers residing closer to the center, we can show that the results remain qualitatively the same.

equilibrium land rent (11) into (10), we obtain

$$V^w = n + I^* - \xi C - tL/2 + \overline{q}_0. \tag{13}$$

From the budget constraint of criminals, (3), we obtain

$$q_0 = \xi L - R^a + \overline{q}_0 - \int_0^n p(v)q(v)\,\mathrm{d}v.$$

By plugging this value and the equilibrium quantities and prices (5) and (6) into the utility (1) and adding the cost c of committing crime, we obtain

$$V^c = n + \xi L - c + \overline{q}_0 \tag{14}$$

Thus, the value of c making a marginal individual indifferent between committing a crime and working is $\tilde{c}$ and is given by

$$\tilde{c} = (\xi - \varphi)\lambda + \frac{t(1-\theta)}{2}\lambda \tag{15}$$

where φ is defined by (9). Hence, because of the uniform distribution of c, the fraction of criminals is $\theta = \tilde{c}$. The equilibrium share of criminals θ^* is thus given by

$$\theta^* = \frac{\lambda t + 2\lambda(\xi - \varphi)}{\lambda t + 2}. \tag{16}$$

It is easily verified that $\theta^* < 1$ if and only if $\lambda(\xi - \varphi) < 1$. A sufficient condition for $\theta^* < 1$ is $\xi < \varphi$. We thus assume throughout that

$$\xi < \varphi. \tag{17}$$

Moreover, $\theta^* > 0$ if and only if

$$t > 2(\varphi - \xi). \tag{18}$$

In this context, it is easily checked that $\partial\theta^*/\partial t > 0$ as soon as $\theta^* < 1$, which is guaranteed by (17). We have the following comparative statics results:

Proposition 1. *Assume that* (17) *and* (18) *hold. Then,*

(i) *worse job access* (*or higher commuting costs*) *leads to more criminal activities in the city*;
(ii) *the impact of commuting costs on criminal activities is higher when the city size increases*;
(iii) *the more differentiated are the products or the lower is the labor productivity*, *the higher is the mass of criminals in the city*;
(iv) *criminal activities increase more than proportionally when the city size increases*.

The first result (i) is due to the fact that when commuting costs increase, the total urban cost also increases so that the net wages of workers are reduced. This, in turn, leads to a larger fraction of individuals committing crime. This implies that a transport policy that aims at improving job access (a lower t) would reduce criminal activities in the short run. We will investigate this issue in more detail in the next section. In addition, because $\partial^2\theta^*/\partial\lambda\partial t > 0$, the impact of commuting costs on criminal activities is higher when there is an increase in city size (result (ii)). This is because urban costs are positively correlated with population size and thus, the effect of commuting costs on land rents is higher in larger cities. Finally, $\partial\theta^*/\partial\gamma > 0$, which means that the mass of criminals decreases with more differentiated products (lower γ). Indeed, when γ decreases, φ increases, meaning that the revenue per worker is higher for firms because there is less price competition and thus workers obtain higher wages, which deter criminality. A similar effect can be found for labor productivity $(1/\phi)$ since $\partial\theta^*/\partial\phi > 0$ (result (iii)).

If we now look at the effect of city size on criminal activities, we see that

$$\frac{\partial\theta^*}{\partial\lambda} = \frac{t - 2(\varphi - \xi)}{1 + t\lambda/2} > 0$$

as long as $\theta^* > 0$, which is the result (iv). Thus, *a larger population in a city triggers more criminals in that city*. This is due to the fact that in bigger cities, people are more induced to be criminals since they experience higher urban costs (land rents and commuting costs) and obtain higher proceeds from crime (see (15)). However, they also obtain a higher wage. We show that the former effect dominates the latter and thus, crime per capita increases with larger cities. This gives a micro-foundation to the empirical result found in Glaeser and Sacerdote (1999). Even if the amount of stolen wealth by each criminal $\xi(1-\theta)\lambda$ is proportional to city size, there is proportionally more crime in bigger cities than in smaller cities.

5. Police resources and crime

In this section, we check whether our main results hold when the city government implements a policy fighting criminal activities. We now consider that the number of active criminals in the city is given by $(1-a)C$, where $a \in [0, 1]$ is the share of

criminals in jail or, equivalently, the individual probability of being caught (by the law of large numbers). This share of active criminals depends on the resources used by the local government of the city to fight criminal activity. We consider that the probability of arresting a criminal is an increasing function of the *per capita* public resources, denoted by T. These public expenditures are financed by a local head tax τ paid by workers. More precisely, we assume that $a \equiv f(T)$, where $T = \tau L/\lambda$, which are the total resources per capita invested in police for the local government. For the same level of tax revenue, the probability of being arrested is lower in a larger city. Increasing the resources devoted to police (higher τ or/and L) and decreasing the population size (λ) raise the probability of arresting a criminal.

In addition, we assume, quite naturally, that $f(0) = 0$, $f'(T) > 0$ and $f''(T) < 0$. Because $L = (1-\theta)\lambda$, we have $T = \tau(1-\theta)$. As a result, if $\theta = 1$, then $f(T) \to 0$, *i.e.* the probability of being arrested tends to zero when there is no worker because there is no public resource. In contrast, if $\theta = 0$, then $T = \tau$, which is its maximum value. In that case, we assume that $f(\tau) \to 1$ when $T \to \tau$, *i.e.* the probability of being arrested is close to 1 when a worker becomes a criminal if there is no criminal in the city.

The timing is as follows. In the first stage, the local government chooses its tax policy τ. In the second stage, types (or honesty parameters c) are revealed and individuals decide to commit crime or not while product, land and labor markets are clear. As usual, the game is solved by backward induction.

5.1. Taxation and share of criminals

Let us solve the stage where crime is decided for a given taxation level τ. Using (14), the indirect utility of a criminal is now given by

$$V^c = n + [1 - f(T)]\xi L - c + \overline{q}_0 \tag{19}$$

with $T = \tau L/\lambda = \tau(1-\theta)$ whereas, using (13), we have

$$V^w = n + I^* - [1 - f(T)]\xi C - tL/2 + \overline{q}_0 - \tau \tag{20}$$

which is the indirect utility of a worker. Indeed, the number of active criminals is $1-a$ since a represents the fraction of criminals in jail (incapacity effect) and $f(T)$ is the probability of being arrested. Note that the effect of the local head tax τ on V^w is ambiguous since there is a direct negative effect (the net income of workers diminishes) and an indirect positive effect through $f(T)$ (more criminals are in jail inducing a higher net income for workers). For simplicity, we assume that only her [the criminal's?] income is affected if she is arrested. We do not consider that her demand for the differentiated service is equal to zero if she is arrested so that we instead have n of $[1 - f(T)]n$ in (19) and I^* instead of $I^* - \varphi f(T)C$ in (20). Indeed, introducing those effects in the analysis increases the complexity while the gains in terms of results are very limited as they are second-order effects.

Using (19) and (20) and the fact that $L = (1-\theta)\lambda$, $L + C = \lambda$, and $I^* = \varphi\lambda$, the value of c, making an individual indifferent between committing a crime and working is now given by

$$\tilde{c} = (\xi - \varphi)\lambda + \frac{t(1-\theta)}{2}\lambda + \tau - \xi f(T)\lambda \equiv \Omega(\theta). \tag{21}$$

Compared to the case with no policy (see (15)), there is now an additional term, $\tau - \xi f(T)\lambda$, which is endogenous and depends on $T = \tau(1-\theta)$. As before, because of the uniform distribution of c, the fraction of criminals is $\theta = \tilde{c}$. The equilibrium share of criminal θ^* is now implicitly defined by

$$\theta^* = \Omega(\theta^*). \tag{22}$$

Lemma 1. *Assume* (17) *and* $\tau < 1$. *Then if*

$$\frac{t}{2} > \max\{\varphi, \xi\tau f'(0)\} \tag{23}$$

there exists a unique θ^* *for which* $0 < \theta^* < 1$.

Proof. See Appendix A.

Compared to the previous case with no taxation, the conditions are somewhat more demanding but still relatively similar since (18) is part of (23).

Lemma 2. *Assume that* (17), $\tau < 1$ *and* (23). *Then, there is an U-shaped relationship between* θ^* *and* τ *and* θ^* *reaches its minimum value when* $\tau = \tau^c > 0$, *which is the unique minimum and* (*implicitly*) *defined by*

$$\frac{\partial\theta^*}{\partial\tau} = \frac{\partial\Omega(\theta)/\partial\tau}{1 - \partial\Omega(\theta)/\partial\theta} = \frac{-\xi\lambda f'(\tau^c(1-\theta))(1-\theta^*) + 1}{1 + t\lambda/2 - \xi\lambda f'(\tau^c(1-\theta))\tau^c} = 0. \tag{24}$$

Proof. See Appendix A.

A rise in the tax rate has an ambiguous effect on the share of criminals in each city. On the one hand, it increases the probability of arresting a criminal so that less individuals have an incentive to become a criminal. On the other hand, it

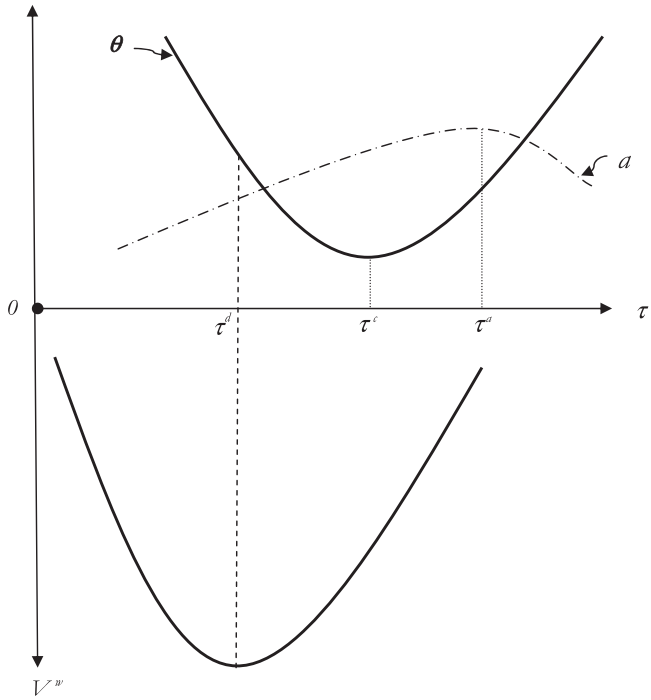

Fig. 3. Tax policy and criminal activity.

directly reduces the legitimate net income for all workers, thus making the criminal activity more attractive. Hence, there is a U-shaped relationship between θ^* and τ (see Fig. 3) and θ^* reaches its minimum value when $\tau = \tau^c$. Indeed, starting from low levels of the tax rate, a higher tax burden reduces the share of criminals in the city. Above the critical value of the tax rate (τ^c), criminal activities rise with the tax burden.

Furthermore, by differentiating (22), we obtain

$$\frac{\partial\theta^*}{\partial\lambda} = \frac{(1-\theta^*)t/2 + [1-f(T)]\xi - \varphi}{1 + t\lambda/2 - \xi\lambda f'(\cdot)\tau} > 0 \quad \text{and} \quad \frac{\partial\theta^*}{\partial t} = \frac{(1-\theta^*)\lambda/2}{1 + t\lambda/2 - \xi\lambda f'(T)\tau} > 0$$

which are both positive when the conditions in Lemma 1 hold. As expected, regardless of tax rates, *the share of criminals in the city is reduced when there is a decrease in the commuting cost* (*or better job access*) *and the city size*. These two results are largely supported by empirical evidence (see Glaeser, 1998; Glaeser and Sacerdote, 1999; Ihlanfeldt, 2002; Olarte Bacares, 2014).

5.2. Tax policy, police and criminal activity

Let us now solve the first stage where the local (urban) government sets a tax rate τ that maximizes the welfare of the representative worker (or, equivalently, the median voter), given by (20). Note that the specification of the government's objective is a controversial issue in our case because individuals can either work or be criminals. We assume that the local government disregards the indirect utility of criminals and only maximizes the welfare of the representative worker. At the end of this section, we discuss an alternative objective.

Raising the tax rate leads to two opposite effects on the welfare of workers. On the one hand, welfare increases as the share of criminals in jail grows. On the other hand, welfare decreases since the tax burden and land rents paid by workers rise. In addition, remember that if the fraction of criminals in jail increases by τ, its effect on the share of criminals in the city is ambiguous. Using $C^* = \theta^*\lambda$, $L^* = (1-\theta^*)\lambda$, $n^* = L^*$ and $I^* = \varphi\lambda$ as well as $V^w = V^c(\tilde{c})$, (20) can be written as

$$V^w = (1-\theta^*)\lambda - \theta^* + \xi\left[1 - f(T^*)\right](1-\theta^*)\lambda$$

where $T^* = \tau(1-\theta^*)$. The first-order condition of the government maximization is then given by

$$\frac{\mathrm{d}V^w}{\mathrm{d}\tau} = -\left\{\lambda + 1 + \xi\left[1 - f(T^*)\right]\lambda\right\}\frac{\partial\theta^*}{\partial\tau} - \xi\lambda(1-\theta^*)f'(T^*)\frac{\mathrm{d}T}{\mathrm{d}\tau}.$$

The equilibrium tax rate is denoted by τ^d and is such that $\mathrm{d}V^w/\mathrm{d}\tau = 0$. Since $\partial\theta^*/\partial\tau = 0$ implies that $\xi\lambda f'(T^*)(1-\theta^*) = 1$, we have

$$\left.\frac{\mathrm{d}V^w}{\mathrm{d}\tau}\right|_{\tau=\tau^c} = -\left.\frac{\mathrm{d}T}{\mathrm{d}\tau}\right|_{\tau=\tau^c} = -(1-\theta^*) < 0.$$

As a result, $\tau^d < \tau^c$ and, when $\tau = \tau^d$, $\partial\theta^*/\partial\tau < 0$. Hence, for a given city size λ, the tax rate that maximizes the utility of the median voter is lower than the tax rate inducing the minimum value of the share of criminals (see Fig. 3) because, at τ^c, taxes are too high. We can also conclude that $\mathrm{d}T/\mathrm{d}\tau > 0$ when $\tau = \tau^d$. Hence, the equilibrium tax rate is in the upward-sloping portion of the curve.

In addition, there exists an interior solution which is positive if and only if

$$\left.\frac{\partial V^w}{\partial\tau}\right|_{\tau=0} = -(1+\lambda+\xi\lambda)\left.\frac{\partial\theta^*}{\partial\tau}\right|_{\tau=0} - \xi f'(0)(1-\theta^*)^2\lambda > 0$$

where $\partial\theta/\partial\tau < 0$ when $\tau = 0$ and $\xi f'(0)(1-\theta^*)\lambda > 1$. As a consequence, $\tau^d > 0$ if and only if

$$\frac{\xi\lambda^2 f'(0)[1-(\xi-\varphi)\lambda](1+2\xi-\varphi)}{1+\lambda(1+\xi)} > 1 + \frac{t\lambda}{2}$$

where $1-(\xi-\varphi)\lambda > 0$. Hence, the local urban government is more likely to levy taxes to fight criminal activities when the commuting costs are low enough. Furthermore, lower commuting costs have a direct impact on the share of criminals in each city and through a change in tax policy. Indeed, we have

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}t} = \frac{\partial\theta^*}{\partial t} + \frac{\partial\theta^*}{\partial\tau}\frac{\partial\tau^d}{\partial t}$$

where $\partial\theta^*/\partial t > 0$ (see Section 5.1) and $\partial\theta^*/\partial\tau$ as $\tau^d < \tau^c$. In Appendix A, we show that $\partial\tau^d/\partial t < 0$ so that $\mathrm{d}\theta^*/\mathrm{d}t > 0$. Hence, the local government adjusts upward its tax rate when commuting costs decline, which leads to less criminals in each city. This means that when commuting costs are low (or job access is good), the optimal tax rate τ^d chosen by the local government is closer to τ^c, *i.e.* the tax rate that minimizes total crime. The intuition of this result is straightforward. Since the incentives for becoming a criminal are lower and the workers' welfare increases when the commuting costs are reduced, the local government can increase its tax rate so that the share of criminals is reduced.

Proposition 2. *Assume (17), $\tau < 1$ and (23). Then, lower commuting costs or better job access makes the impact of police on crime more efficient.*

Proof. See Appendix A.

We could consider the case where the government aims at maximizing the probability of arresting a criminal (a). In that case, the tax rate that maximizes the probability of arresting a criminal τ^a is implicitly given by

$$\frac{\mathrm{d}a}{\mathrm{d}\tau} = f'(\cdot)\frac{\mathrm{d}T}{\mathrm{d}\tau} = f'(\cdot)\left(1-\theta^* - \tau\frac{\partial\theta^*}{\partial\tau}\right) = 0$$

and is identical to the tax rate maximizing the tax revenue. Starting from a low tax rate, higher tax rates raise the public resources per capita and, in turn, increase the probability of arresting a criminal (see Fig. 3). Beyond τ^a, a rise in the tax rate reduces the revenue from the tax because the number of taxpayers (workers) reaches low values (a variant of the Laffer curve).[16] It is straightforward to check $\mathrm{d}a/\mathrm{d}\tau > 0$ when $\tau = \tau^c$ so that $\tau^a > \tau^c$. Hence, *a local government maximizing the public resources to fight criminal activity induces more criminals and a larger tax burden than a local authority minimizing the number of criminals*.

6. Criminal activities, migration and urbanization

In this section, we extend our model to two cities and verify whether our main results hold when individuals can migrate between cities. There are two cities: city A with a population size given by λ_A and city B with a population size given by λ_B. Without loss of generality, we assume that $\lambda_A + \lambda_B = 1$ (the total population in the economy is normalized to one) and $\lambda_A \geq \lambda_B$. To facilitate the notation, we set $\lambda_A = \lambda$ and $\lambda_B = 1-\lambda$, with $\lambda \geq 1/2$.

Let us now endogenize the location choice of all individuals (λ). For simplicity, we return to the benchmark model with no police so that $a = 0$ in each city. The timing of the model is now as follows. In the first stage, households choose in which city they will reside without knowing their type c but anticipating (with rational expectations) the average total population of criminals. Hence, the location choice of individuals is based on the difference in *expected utilities* across cities. Indeed, we assume that types are revealed after location choices have been made to take into account the relative inertia of the land market compared to the crime and labor markets (see Verdier and Zenou, 2004, for a similar assumption). Individuals make

[16] Note that $\mathrm{d}^2a/\mathrm{d}\tau^2$ when $\tau = \tau^a$.

quicker decisions in terms of crime or labor than in terms of residential location. In the second stage, types (or honesty parameters) are revealed and individuals decide whether or not to commit crime. In the third stage, goods are produced, workers participate in the labor market while criminals participate in the crime market and all consume the two types of goods. Observe that in the second stage, workers are stuck in their initial locations (determined in the first stage) and cannot relocate to another city. They then decide whether to become criminal or not. Since we have solved the last two stages in the previous sections, let us determine the location choice of individuals.

6.1. Location choices

The location of individuals is driven by the inter-city difference in their expected utility. Before knowing their c, the expected utility of living in city $r=A,B$ is given by (using (13) and (14)):

$$\mathbf{EV}_r = \int_0^{\tilde{c}_r} V_r^c \, \mathrm{d}c + \int_{\tilde{c}_r}^1 V_r^w \, \mathrm{d}c = \theta_r^2/2 + V_r^w(\theta_r) \tag{25}$$

where $\tilde{c}_r = \theta_r$ and, according to (16), θ_r is given by

$$\theta_r^* = \frac{\lambda_r t + 2\lambda_r(\xi - \varphi)}{\lambda_r t + 2}.$$

As in Section 4, we assume throughout this section that conditions (17) and (18) hold to guarantee that there exists a unique $0 < \theta_r^* < 1$ in each city $r=1, 2$. Observe that the expected utility (25) is based on θ_r, the average proportion of criminals in city r. Also observe that even though the individual demands of product variety v (5) are unaffected by income, the migration decision takes income into account. Indeed, everything else equal, workers are drawn by the city with the higher wage. When the population becomes larger, the local demand for services rises, which attracts additional firms. In addition, households are attracted by larger cities in order to have access to more varieties (taste for diversity; see (1)). This implies a rise in the wage rate. On the other hand, competition for land among workers raises land rent and commuting costs, in which both increase with population size. In addition, the location of households is affected by the level of criminal activity (in accordance with empirical evidence; see Cullen and Levitt, 1999). These different mechanisms interact with the decision to become a criminal and, in turn, the level of wages, urban costs, and agglomeration. In other words, our model captures *general-equilibrium effects*.

Hence, the spatial difference in the expected utility $\mathbf{EV}_A - \mathbf{EV}_B \equiv \Delta\mathbf{EV}$ is given by

$$\Delta\mathbf{EV}(\lambda, \theta_A, \theta_B) = \left(\lambda - \frac{1}{2}\right)\Gamma(\lambda, t) \tag{26}$$

where

$$\Gamma(\lambda, t) \equiv \frac{-t(1+\xi)(\xi-\varphi)(1-\lambda)\lambda + 2[1-\xi(1+\xi+\varphi)+2\varphi]}{(1+\lambda t/2)[1+(1-\lambda)t/2]} + \frac{2(\xi-\varphi+t/2)^2[t(1-\lambda)\lambda+1]}{(1+\lambda t/2)^2[1+(1-\lambda)t/2]^2}.$$

We would now like to analyze the equilibrium of this economy, which is defined so that no individual (worker or criminal) has an incentive to change location (or city).

Definition 1.

(i) An equilibrium arises at $0 < \lambda^* < 1$ when the utility differential $\Delta\mathbf{EV}[\lambda^*, \theta_r(\lambda^*)] = 0$, or at $\lambda^* = 1$ when $\Delta\mathbf{EV}[1, \theta_r(1)] \geq 0$ or at $\lambda^* = 0$ when $\Delta\mathbf{EV}[0, \theta_r(0)] \leq 0$.
(ii) An interior equilibrium is stable if and only if the slope of the indirect utility differential $\Delta\mathbf{EV}$ is strictly negative in a neighborhood of the equilibrium, *i.e.*, $\mathrm{d}\Delta\mathbf{EV}[\lambda^*, \theta_r(\lambda^*)]/\mathrm{d}\lambda < 0$ at λ^*.
(iii) A fully agglomerated equilibrium (*i.e.* when $\lambda^* = 1$ or $\lambda^* = 0$) is stable whenever it exists.

It is well-known that new economic geography (NEG) models typically display several spatial equilibria (Combes et al., 2008; Fujita and Thisse, 2013). In such a context, it is convenient to use stability as a selection device since an unstable equilibrium is unlikely to occur. This is what is exposed in Definition 1 where an interior equilibrium is stable if, for any marginal deviation away from the equilibrium, the incentive system provided by the market brings the distribution of individuals back to the original one. In (ii), we give the conditions for which the equilibrium is stable.

The analysis of an existence and stability equilibrium is reported in Appendix B. We show that full dispersion is more likely to occur when (i) commuting costs are high enough (like in NEG models, see Gaigné and Thisse, 2014) or (ii) when the amount stolen by criminals (ξ) is high enough $\xi \geq \overline{\xi} \equiv \sqrt{3+4\varphi+\varphi^2} - 1$ (the negative externality created by criminal activity is high). In contrast, full agglomeration occurs when commuting costs reach low values if $t < \underline{t}(\xi)$, where

$$\underline{t}(\xi) \equiv 2\left[-\xi^2 + \xi\varphi + \varphi + (1+\varphi-\xi)\sqrt{1+(1+\xi)^2}\right]. \tag{27}$$

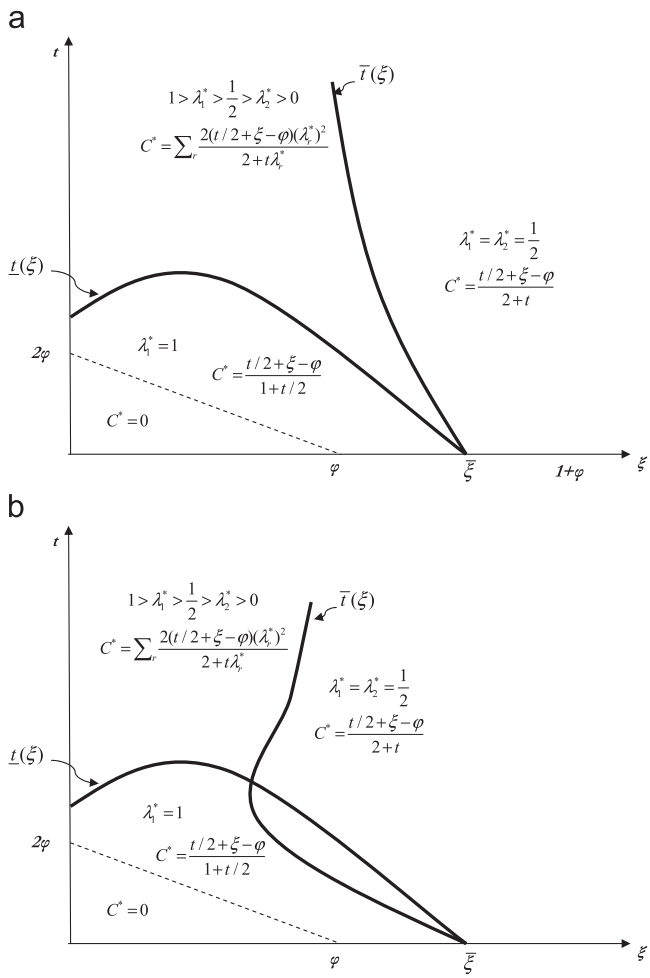

Fig. 4. Spatial equilibria and crime in (t,ξ) -space: (a) high $\varphi(\varphi>1)$ and (b) low $\varphi(\varphi<1)$.

In Fig. 4, we have depicted $\underline{t}(\xi)$, which is a non-linear curve that increases and then decreases up to $\underline{t}(\xi)=0$. It appears that the same set of parameters may yield two stable spatial equilibria ($\lambda_r^*=1/2$ or $\lambda_r^*=1$ when $\bar{t}(\xi)<t<\underline{t}(\xi)$, see Fig. 4b). In other words, different levels of criminal activity may emerge for the same economic conditions.

Let us now study *partial agglomeration* ($\lambda^*\in(1/2,1)$), which occurs when $\Gamma(\lambda^*)=0$. This is the case when $t>\underline{t}(\xi)$. In Appendix B, we show that an *asymmetric spatial configuration* emerges when commuting costs take intermediate values. It appears that $\underline{t}(\xi)>2(\varphi-\xi)$ for all $t,\xi>0$, which guarantees that $\theta_r^*>0$ where there is no full agglomeration. In addition, under this spatial configuration, we have

$$L_A^*-L_B^*=\left(\lambda^*-\tfrac{1}{2}\right)\Lambda(\lambda^*)$$

where

$$\Lambda\equiv\frac{2(1-\xi+\varphi)+t(\varphi-\xi)(1-\lambda^*)\lambda^*}{(1+t\lambda^*/2)[1+t(1-\lambda^*)/2]}$$

and where $L_A^*=L_B^*$ when $\lambda^*=1/2$ and $L_A^*>L_B^*$ when $\lambda^*>1/2$ because $\partial\Lambda(\lambda)/\partial\lambda>0$. In other words, the large city hosts more workers and more criminals. It is also worth stressing that, *ex post*, *workers living in the smaller city are better off than workers living in the larger city* (*ex ante* they all have the same expected utility). Indeed, because $\theta_A^*>\theta_B^*$ and $\Delta\mathbf{EV}(\lambda)=0$ when $1/2<\lambda^*<1$, then $V_A^w<V_B^w$.

The following proposition summarizes all our main findings whereas Fig. 4 displays the spatial equilibria in the $t-\xi$ space.

Proposition 3. *There are three stable spatial equilibria with respect to commuting costs* (*or job access*):

(i) *If $\Gamma(1/2,t)<0$, i.e. the commuting costs are high enough, there are two identical cities in population size, $\lambda_A^*=\lambda_B^*=1/2$, and in the share of criminals, $\theta_1^*(1/2)=\theta_2^*(1/2)$.*

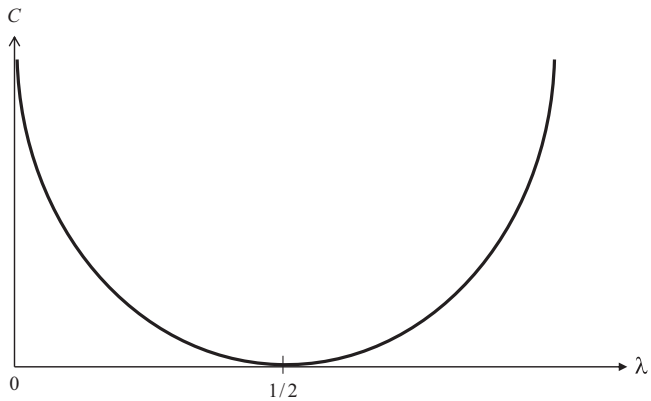

Fig. 5. Relationship between total crime and agglomeration.

(ii) *If $\Gamma(1/2,t)>0$ and $t>\underline{t}$, i.e. commuting costs take intermediate values, there is a large city and a small city where the former has more criminals and more workers than the latter, $1/2<\lambda_r^*<1$, $L_1>L_2$ and $\theta_1^*>\theta_2^*$.*

(iii) *If $t<\underline{t}$, i.e. commuting costs are low enough, there is a single city.*

6.2. The impact of commuting costs on criminal activities

We now analyze the impact of commuting costs (t) on the criminal activity when there is *free mobility* of workers between the two cities and see if our previous results hold. As above, a decrease in t can be interpreted as a more efficient transport policy or better access to jobs. We can now define the mass of criminals in the economy as a function of the relative size of cities λ. It is given by

$$C(\lambda)=\lambda\theta_A^*+(1-\lambda)\theta_B^*=\frac{[t+2(\xi-\varphi)][(4-t)\lambda^2-(4-t)\lambda+2]}{(2+\lambda t)[2+(1-\lambda)t]}. \tag{28}$$

It is easily verified that $\partial C(\lambda)/\partial\lambda>0$ as long as $\lambda\geq 1/2$. This means that when the size of the population in the first city is more than 50 percent, then total crime in the economy increases by λ. There is a U-shape relationship between total crime C and λ as illustrated in Fig. 5. In our model, agglomeration is defined by $\lambda\neq 1/2$ and the further away is λ from 1/2, the more agglomeration there is.

We have seen in Section 4 that lower commuting costs (better job access) reduce total criminal activities when the location choice of individuals is exogenous (*i.e.* when λ was given). This is no longer true when individuals choose location and, in fact, the total effect is ambiguous. Indeed, at any given location of households, lower commuting costs reduce the number of criminals in each city ($\partial C_r/\partial t>0$) because $\partial\theta_r/\partial t>0$ regardless of city r. On the other hand, the location of individuals adjusts to a change in commuting costs in the long run. More precisely, falling commuting costs promote agglomeration ($\partial\lambda/\partial t<0$) and, in turn, more crimes are committed in the larger city ($\partial C_A/\partial\lambda>0$) while the number of crime in the small city shrinks ($\partial C_B/\partial\lambda<0$). As a result, the long-run effect associated with a decrease in commuting costs on criminal activity is ambiguous. Even though lower commuting costs induce a higher legitimate net income for all workers, they also promote higher levels of agglomeration. This is a new and interesting result which shows that the impact of job access or transportation policy on crime differs between the short and long run.

Let us investigate this issue in more detail. Consider first that the economy shifts from full dispersion to full agglomeration due to lower commuting costs. Under these spatial configurations, we have

$$C(1/2)=\frac{t/2+\xi-\varphi}{2+t/2}\text{ for }\Gamma(1/2,t)<0\quad\text{and}\quad C(1)=\frac{t/2+\xi-\varphi}{1+t/2}\text{ for }t<\underline{t}.$$

For example, it appears that $C(1/2,t=\bar{t}_2)>C(1,t=\underline{t})$ if and only if $\varphi>\widehat{\varphi}$ where

$$\widehat{\varphi}\equiv\frac{2(5+3\xi)\sqrt{2\xi+\xi^2+2}-5\xi^2-15\xi-14}{1+\xi}$$

and $\widehat{\varphi}$ is positive and increases with ξ. Hence, a shift from dispersion to agglomeration due to lower commuting costs may give rise to a decline in criminal activity. The final effect is that there are less criminal activities in the economy (the former effect dominates the latter effect).

In addition, $C(\lambda)$ reaches its minimum value when $t\leq t_{\min}$ ($C=0$). It is easily verified that $t_{\min}<\underline{t}$ so that $C=0$ may occur when $\lambda^*=1$ and not when $\lambda^*=1/2$.[17] Therefore, improving the access to jobs by reducing the commuting costs can be a relevant policy tool in reducing crime.

[17] Indeed, $\Delta(1)=1+\xi$ when $t=t_{\min}$ and $\mathrm{d}\Delta\mathbf{EV}(\lambda^*)/\mathrm{d}\lambda>0$ at $\lambda^*=1/2$ when $t=t_{\min}$.

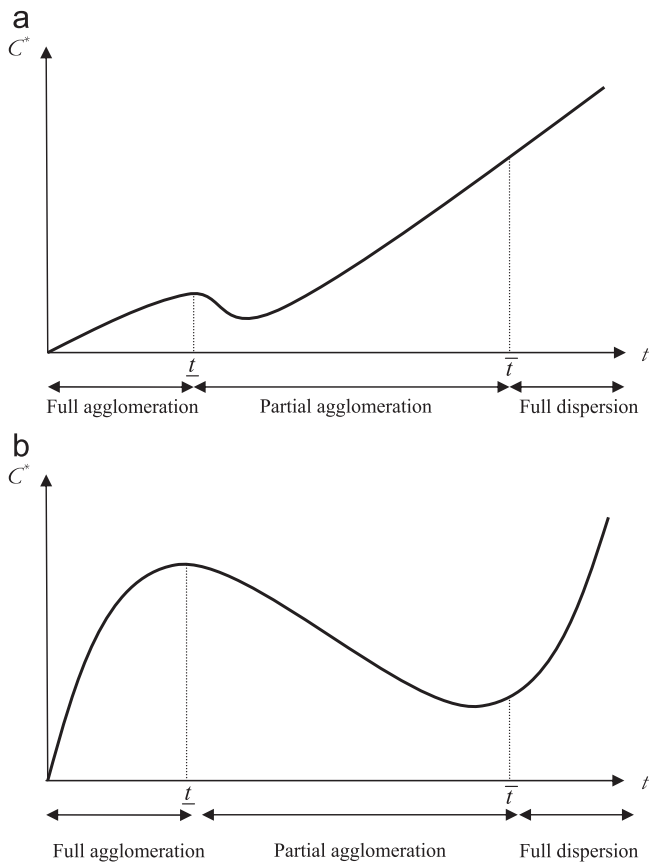

Fig. 6. The impact of commuting costs on criminal activities: (a) high $\varphi(\varphi > \hat{\varphi})$ and (b) low $\varphi(\varphi < \hat{\varphi})$.

In the case when t decreases when partial agglomeration occurs, the degree of agglomeration (λ^*) increases gradually so that the relationship between $C(\lambda^*)$ and t is ambiguous when $\overline{t} > t > \underline{t}$. Because λ^* is highly nonlinear, we need to perform some numerical simulations. These simulations reveal a U-shaped relationship between $C(\lambda^*)$ and t. There exists a threshold value $\widehat{t}$ such that $C(\lambda^*)$ decreases with a reduction in commuting costs when $\overline{t} > t > \widehat{t}$. However, notwithstanding criminal activity may fall in the economy when t moves from $\overline{t}$ to $\underline{t}$, crime increases occur in the larger city due to a larger population size. Fig. 6 displays the relationship between crime and commuting costs.

To summarize:

Proposition 4. *When there is no mobility between cities* (*short run*), *decreasing commuting costs* (*or improving job access*) *always increases total crime. When there is free mobility between the two cities* (*long run*), *a negative relationship between commuting costs and total crime is more likely to occur if* φ *is low and* ξ *is high* ($\varphi < \widehat{\varphi}$).

Indeed, when t decreases, there will be more agglomeration, which leads to two opposite effects. On the one hand, the urban costs in the big city will increase as compared to the small city and thus, more people decide to become criminal. On the other hand, real wages increase in the big city because of a bigger market size, which reduces the number of criminals. The net effect is thus ambiguous. When φ is high and ξ is low, the latter effect dominates the former for a large range of commuting costs while we have the reverse result, when φ is low and ξ is high. Remember that the incentive to become a criminal is relatively strong when real labor productivity (φ) is low or crime productivity (ξ) is high.

7. Concluding remarks

Summary: This paper provides the first model of agglomeration and crime in a general equilibrium framework. We first develop a one-city model where individuals are heterogeneous in their incentives to commit crime, freely choose their location within the city and decide whether to become a criminal or a worker. We show that higher commuting costs lead to more criminal activities in the city and that the impact of commuting costs on criminal activities is higher when there is an increase in city size. We also show that the mass of criminals decreases with more differentiated products and with higher

labor productivity and that criminal activities increase more than proportionally when the city size increases. We also consider an urban government that levies a tax on workers in order to finance policy resources to fight criminal activities. We show that there is a U-shaped relationship between the crime rate in the economy and the tax rate so that increasing resources to fight crime financed by taxes on workers can backfire if the tax rate is too high. We also show that lower commuting costs or better job access make the impact of police on crime more efficient.

Then, we extend our framework to allow individuals to freely move between two cities. We totally characterize the three different stable equilibria as a function of commuting costs. When the commuting costs are high enough, the population of workers and criminals is evenly distributed between the two cities. When the commuting costs take intermediate values, there is a large and a small city where in the former there are more criminals and workers than in the latter. When the commuting costs are low enough, there is a single city. We also show that criminal activity increases in the economy when commuting costs decline only if the size of the agglomeration is high enough. Finally, while we have shown that in the short run (only one city) better accessibility to jobs decreases crime, we show that, in the long run (free mobility between the two cities), this policy may, on the contrary, increase crime. This means that policy makers should be careful in implementing a transportation policy because it may backfire on crime. Our paper provides an explanation for why this might be the case.

Policy discussion: In this paper, we mainly focus on a key policy instrument which is travel cost. This is because, in standard monocentric models (Fujita, 1989) and in their multicentric extensions (Fujita and Thisse, 2013), unit travel cost is usually the fundamental parameter that determines the location choices of households within cities, their consumption of housing, land use and the population size of cities. Moreover, as highlighted by Rietveld (2005), transport is characterized by market failures that are of particular importance in urban settings so policies related to transportation are crucial to implement in cities. Here, we have mainly focused on the impact of transportation policies on crime. However, it is clear that transportation policies that reduce commuting costs in the city also have a direct impact on the outcomes mentioned above such as, for example, the location choices of households within cities and the population size of cities. For example, it has been shown that radial highways have a positive impact on the decentralization of the population from central cities to suburbs (9% per ray for US cities; see Baum-Snow, 2007). Highways also cause a dramatic increase in driving, and highways and railroads cause an increase in economic activity in rural areas near highways (Redding and Turner, 2015). As a result, implementing a transportation policy (by, for example, constructing a new highway) has an impact on crime (as shown in this paper) but also has an impact on other aspects of urban life, which we did not consider here.

We also consider another policy, which is the impact of taxes on crime (Section 5). In fact, this last policy consists of increasing the number of policemen in a city because there is a direct positive relationship between taxes and the probability of being arrested for criminals. Taxes are levied on workers' income. We first show that increasing taxes and thus the number of policemen in the city have an ambiguous effect on the crime rate because, on the one hand, it directly deters crime because criminals are more likely to be arrested but, on the other, it induces more people to become criminals since the net income of workers decreases. In Proposition 2, we show that when travel costs are low (for example, because of a policy that improves transport), then increasing the number of policemen in a city (by increasing taxes) has a higher negative impact on crime as compared to the case when travel costs are higher. This means that the two policies (transportation and police) are complementary in fighting crime and should be implemented together.

Finally, as mentioned in the Introduction, the quantitative effects of commuting costs on crime are important.[18] Studying the city of Bogotá, Columbia, Olarte Bacares (2014) shows that the introduction of a Bus Rapid Transport (BRT) system called Transmilenio (TM) leads to an increase in crime in areas that benefit from the TM. Indeed, the number of "thefts from people" committed in zones that benefit from TM was, on average, 133 while the average number of thefts from people in zones without TM was 78. The biggest difference in thefts committed between these two kinds of zones in 2007 was approximately 54. Those results are statistically significant at the 1% level. Similar results were found for the number of robberies against people with violent aggression and the number of house breakings. Ihlanfeldt (2002, 2006, 2007) shows that for the city of Atlanta in the United States, the job access effects on crime are statistically significant in black neighborhoods across all crime categories, but in white neighborhoods this is only true for property crimes. These results suggest that when legitimate jobs are not located nearby, black youth are more likely than white youth to turn to crime. The economic significance of the results is best illustrated by the finding that the racial difference in neighborhood property crime rates would be 21% smaller if black youth enjoyed the same access to jobs as white youth. The corresponding percentages for violent crime and drug abuse violations are 5.5 and 13.7%, respectively. These last results show the importance of job accessibility on crime, especially for black youth. Transportation policies usually facilitate job access by reducing commuting costs. These effects are important and we believe that more empirical studies on these issues should be conducted following the results of our theoretical model. We leave that to future research.

[18] Recent research (see, in particular, Duranton and Turner, 2011, 2012; Couture et al., 2013) provides precise estimates for travel cost and show empirically how it varies with population and road infrastructure.

Appendix A. Proofs of Lemmas 1 and 2 and Proposition 2

Proof of Lemma 1. In (21), we have defined $\Omega(\theta)$ as

$$\Omega(\theta) \equiv (\xi-\varphi)\lambda + \frac{t(1-\theta)}{2}\lambda + \tau - \xi f(T)\lambda.$$

Remembering that $T=\tau(1-\theta)$, it is easily verified that[19]

$$\Omega(0) = \left(\frac{t}{2}-\varphi\right)\lambda + \tau.$$

We can also show that

$$\Omega(1) = (\xi-\varphi)\lambda + \tau$$

and

$$\Omega'(\theta) = \left(\tau\xi f'(T) - \frac{t}{2}\right)\lambda$$

$$\Omega''(\theta) = -\lambda\tau^2\xi f''(T) > 0.$$

Let us show that there exists a unique θ^*, strictly between 0 and 1, which is defined as $\theta^* = \Omega(\theta^*)$. For that, we will give conditions guaranteeing that (i) $\Omega'(\theta) < 0$, (ii) $\Omega(0) > 0$ and (iii) $\Omega(1) < 1$.

(i) $\Omega'(\theta) < 0$: For this to be true, it has to be that $t/2 > \tau\xi f'(T)$. Since $f'(T)$ is a decreasing function, a sufficient condition is

$$\frac{t}{2} > 2\tau\xi f'(0)$$

which is included in (23).

(ii) $\Omega(0) > 0$: For this to be true, it has to be that $t/2 + \tau/\lambda > \varphi$. Since $\lambda > 0$, a sufficient condition is

$$\frac{t}{2} > \varphi$$

which is included in (23).

(iii) $\Omega(1) < 1$: For this to be true, it has to be that $(\xi-\varphi)\lambda + \tau < 1$. We have assumed throughout that (17) holds, *i.e.* $\xi < \varphi$. Thus, this condition can be written as $\tau < 1 + \lambda(\varphi - \xi)$. Since $\tau > 0$, a sufficient condition is

$$\tau < 1$$

which is included in the conditions given in Lemma 1. □

Proof of Lemma 2. The crime rate is given by (22) where

$$\theta^*(0) = \frac{(t/2+\xi-\varphi)\lambda}{1+t\lambda/2} > 0.$$

By differentiating (22), we easily obtain

$$\frac{\partial\theta^*}{\partial\tau} = \frac{-\xi\lambda f'(T)(1-\theta)+1}{1+t\lambda/2-\xi\lambda f'(T)\tau} = 0$$

which is (24). Denote by

$$N \equiv -\xi\lambda f'(T)(1-\theta)+1$$

and by

$$D \equiv 1 + t\lambda/2 - \xi\lambda f'(T)\tau.$$

Observe that

$$\left.\frac{\partial\theta^*(\tau)}{\partial\tau}\right|_{\tau=0} < 0$$

[19] Remember that we have assumed that

$$\lim_{T\to\tau} f(T) = 1.$$

$$\frac{\partial^2\theta^*}{\partial\tau^2}=\frac{\left[-\xi\lambda^3 f''(T)(1-\theta)^2+\xi\lambda^2 f'(T)\frac{\partial\theta^*}{\partial\tau}\right]D+\xi\lambda^2\left[f''(T)(1-\theta)\lambda\tau+f'(T)\right]N}{D^2}.$$

We look for a minimum and thus $\frac{\partial\theta^*}{\partial\tau}=0$, which implies that $N=0$ and $\frac{\partial\theta^*}{\partial\tau}=0$. Thus,

$$\frac{\partial^2\theta^*}{\partial\tau^2}=\frac{-\xi\lambda^3 f''(T)(1-\theta)^2}{D}>0$$

and there is a unique minimum that we denote by $\tau=\tau^c$.

Observe that τ^c is defined by

$$-\xi\lambda f'(\tau^c L)(1-\theta)+1=0$$

which is equivalent to

$$\tau^c=\frac{f'^{-1}}{L}\frac{1}{\xi\lambda(1-\theta)}>0$$

which proves that $\tau^c>0$. □

Proof of Proposition 2. Note that

$$\frac{\mathrm{d}\tau^d}{\mathrm{d}t}=\frac{\partial\tau^d}{\partial t}+\frac{\partial\tau^d}{\partial\theta}\frac{\partial\theta^*}{\partial t}$$

where the second term is negative as $\partial\theta^*/\partial t>0$ (see Section 5.1) and $\partial\tau^d/\partial\theta<0$ because we have shown that $\tau^d<\tau^c$. In what follows, we show that $\partial\tau^d/\partial t<0$ (for a given θ) so that the sign of $\mathrm{d}\tau^d/\mathrm{d}t$ is always negative.

Using the implicit function theorem yields

$$\left.\frac{\partial\tau^d}{\partial t}\right|_{\mathrm{d}\theta=0}=-\left(\frac{\partial^2 V^w}{\partial\tau^2}\right)^{-1}\left.\frac{\mathrm{d}^2 V^w}{\mathrm{d}t\mathrm{d}\tau}\right|_{\mathrm{d}\theta=0}$$

so that the sign $\partial\tau^d/\partial t$ is given by the sign of $\left.\frac{\mathrm{d}^2V^w}{\mathrm{d}t\mathrm{d}\tau}\right|_{\mathrm{d}\theta=0}$ because $\frac{\partial^2V^w}{\partial\tau^2}$ is negative (the condition which guarantees that τ^d is an interior solution).

To study the sign of $\left.\frac{\mathrm{d}^2V^w}{\mathrm{d}t\mathrm{d}\tau}\right|_{\mathrm{d}\theta=0}$, we first rewrite the first-order condition of government maximization as follows:

$$\frac{\mathrm{d}V^w}{\mathrm{d}\tau}=(B\tau-A)\frac{\partial\theta^*}{\partial\tau}-B(1-\theta^*)=0$$

with $A\equiv\lambda+1+\xi\left[1-f(T^*)\right]\lambda>0$ and $B\equiv\xi\lambda(1-\theta^*)f'(T^*)>0$ where we have used $\mathrm{d}T/\mathrm{d}\tau=(1-\theta^*)-\tau(\partial\theta^*/\partial\tau)$. Hence, we have

$$\left.\frac{\mathrm{d}^2V^w}{\mathrm{d}t\mathrm{d}\tau}\right|_{\mathrm{d}\theta=0}=(B\tau-A)\left.\frac{\partial^2\theta^*}{\partial t\partial\tau}\right|_{\mathrm{d}\theta=0}.$$

Because $(B\tau-A)(\partial\theta^*/\partial\tau)=B(1-\theta^*)>0$ and $\partial\theta^*/\partial\tau<0$ in equilibrium, we get

$$B\tau-A<0.$$

In addition, as

$$\frac{\partial\theta^*}{\partial\tau}=\frac{-\xi\lambda f'(T)(1-\theta^*)+1}{1+t\lambda/2-\xi\lambda f'(T)\tau},$$

we get

$$\left.\frac{\partial^2\theta^*}{\partial t\partial\tau}\right|_{\mathrm{d}\theta=0}=\frac{-\lambda/2}{1+t\lambda/2-\xi\lambda f'(T)\tau}\frac{\partial\theta^*}{\partial\tau}>0$$

because $1+t\lambda/2-\xi\lambda f'(T)\tau$ is positive and $\partial\theta^*/\partial\tau<0$ as $\tau^d<\tau^c$. As a result,

$$\left.\frac{\mathrm{d}^2V^w}{\mathrm{d}t\mathrm{d}\tau}\right|_{\mathrm{d}\theta=0}<0$$

so that $\partial\tau^d/\partial t<0$. □

Appendix B. Existence and stability of equilibrium spatial configurations with a system of two cities

We investigate all the possible equilibria in more detail. First, *full dispersion* ($\lambda^*=1/2$) is always an equilibrium whatever the value of t since $\Delta\mathbf{EV}(1/2)=0$. Second, there is an equilibrium with *full agglomeration* ($\lambda^*=0$ or $\lambda^*=1$) if and only if

$\Delta\mathbf{EV}(1)>0$ and $\Delta\mathbf{EV}(0)<0$. Using (26), it is easily checked that these two conditions are satisfied if and only if $t<\underline{t}(\xi)$, where

$$\underline{t}(\xi)\equiv 2\left[-\xi^2+\xi\varphi+\varphi+(1+\varphi-\xi)\sqrt{1+(1+\xi)^2}\right]$$

which is (27). In Fig. 5, we have depicted $\underline{t}(\xi)$, which is a non-linear curve that increases and then decreases up to $\underline{t}(\xi)=0$ for which $\xi=\overline{\xi}\equiv\sqrt{3+4\varphi+\varphi^2}-1$, with $\overline{\xi}\in(\varphi,\varphi+1)$. Notice that $\underline{t}(\xi)>2(\varphi-\xi)$ for all $t,\xi>0$, which guarantees that $\theta_r^*>0$ where there is no full agglomeration. Let us now study *partial agglomeration* ($\lambda^*\in(1/2,1)$), which occurs when $\Gamma(\lambda^*)=0$. This is the case when $t>\underline{t}(\xi)$ so that we have $\Delta\mathbf{EV}(1)<0<\Delta\mathbf{EV}(0)$.

Let us now look at the stability of the interior equilibria since a full-agglomeration equilibrium is always stable when it exists (see Definition 1). An interior solution $\lambda=\lambda^*$ is stable if and only if

$$\left.\frac{\mathrm{d}\Delta\mathbf{EV}}{\mathrm{d}\lambda}\right|_{\lambda=\lambda^*}=\Gamma(\lambda^*,t)+\left(\lambda^*-\frac{1}{2}\right)\frac{\mathrm{d}\Gamma}{\mathrm{d}\lambda}<0$$

We have two types of interior solutions: *full dispersion* with $\lambda^*=1/2$ and *partial agglomeration* with $\lambda^*\in(1/2,1)$.

Stability for a full dispersion equilibrium: We determine the conditions under which a full dispersion equilibrium ($\lambda^*=1/2$) is a *stable* equilibrium (or equivalently $\Gamma(1/2,t)<0$). Let $\Gamma(1/2,t)\equiv\left.\frac{\mathrm{d}\Delta\mathbf{EV}}{\mathrm{d}\lambda}\right|_{\lambda=1/2}$ be the slope of $\Delta\mathbf{EV}$ at $\lambda=1/2$ where

$$\Gamma(1/2,t)=\frac{(1+\xi)(\varphi-\xi)t^2+4[3\xi(\varphi-\xi)-(2-\varphi-\xi)]t-16(\xi-\overline{\xi})(\xi+2+\overline{\xi})}{4(1+t)^3}$$

with

$$\overline{\xi}=\sqrt{3+4\varphi+\varphi^2}-1\in(\varphi,\varphi+1)$$

where $\Gamma(1/2,0)=4(\xi-\overline{\xi})(\xi+2+\overline{\xi})$ and

$$\left.\frac{\mathrm{d}\Gamma(1/2,t)}{\mathrm{d}t}\right|_{t=0}=(7+3\varphi)\left[\xi-\frac{3\varphi+8}{3\varphi+7}(\varphi+1)\right]<0 \tag{29}$$

which is negative as long as $\xi<\varphi+1$. In addition, $\Gamma(1/2,t)=0$ when $t=\overline{t}_1$ or $t=\overline{t}_2$ with

$$\overline{t}_1\equiv 2\frac{(2-\varphi-\xi)-3\xi(\varphi-\xi)+(2+\varphi-\xi)\sqrt{(1+\xi)^2+4(\xi^2-\xi\varphi-\varphi)}}{(1+\xi)(\varphi-\xi)} \tag{30}$$

$$\overline{t}_2\equiv 2\frac{(2-\varphi-\xi)-3\xi(\varphi-\xi)-(2+\varphi-\xi)\sqrt{(1+\xi)^2+4(\xi^2-\xi\varphi-\varphi)}}{(1+\xi)(\varphi-\xi)} \tag{31}$$

where $2+\varphi-\xi>0$ when $\xi<1+\varphi$, $\overline{t}_2=0$ when $\xi=\overline{\xi}$ and $\overline{t}_2>0>\overline{t}_1$ when $\xi>\varphi$.

As a result:

(i) When $\xi\geq\overline{\xi}$, the function $\Gamma(1/2,t)$ is concave with $\Gamma(1/2,0)\leq 0$ and $\Gamma(1/2,t)<0$ as long as $t>0$. Indeed, if $t=0$ then $\Gamma(1/2,t)\leq 0$ (when $\xi\geq\overline{\xi}$) and d $\Gamma(1/2,t)/$ d $t<0$ when $t=0$. By implication, $\lambda^*=1/2$ is a stable spatial configuration regardless of $t\geq 0$ when $\xi\geq\overline{\xi}$.

(ii) When $\overline{\xi}>\xi>\varphi$, the function $\Gamma(1/2,t)$ is still concave where we now have $\Gamma(1/2,0)>0$ and $\overline{t}_2>0>\overline{t}_1$. Thus, there exists a single positive value of t ($\overline{t}_2$) such that $\Gamma(1/2,\overline{t}_2)=0$ and $\Gamma(1/2,t)<0$ if and only if $t>\overline{t}_2$. Hence, when $\overline{\xi}>\xi>\varphi$, $\lambda^*=1/2$ is a stable spatial configuration if and only if $t>\overline{t}_2$.

(iii) When $\xi=\varphi$, there exists a single positive value of t ($4(1+\varphi)/(1-\varphi)$) such that $\Gamma(1/2,t)=0$ and $\Gamma(1/2,t)<0$ if and only if $t>4(1+\varphi)/(1-\varphi)$. Hence, when $\xi=\varphi$, $\lambda^*=1/2$ is a stable spatial configuration if and only if $t>\overline{t}_3$ with

$$\overline{t}_3\equiv\frac{4(1+\varphi)}{1-\varphi}.$$

(iv) When $\varphi>\xi$, we have $\overline{t}_1>\overline{t}_2$ and the function $\Gamma(1/2,t)$ becomes convex. Hence, $\Gamma(1/2,t)<0$ if and only if $\overline{t}_1>t>\overline{t}_2$. Note that $\overline{t}_2>0$ if and only if $\xi>\max\{\underline{\xi},0\}$ with

$$\underline{\xi}\equiv\frac{2\varphi}{5}-\frac{1}{5}+\frac{2}{5}\sqrt{\varphi^2+4\varphi-1}$$

where $\underline{\xi}=\varphi$ when $\varphi=1$ and $\underline{\xi}<\varphi$ if and only if $\varphi<1$. Hence, $\lambda^*=1/2$ is a stable spatial configuration if and only if $\overline{t}_1>t>\overline{t}_2$ and $\varphi>\xi>\max\{\underline{\xi},0\}$. Note that $\varphi>\max\{\underline{\xi},0\}$ if and only if $\varphi<1$. Hence, $\lambda^*=1/2$ is not a spatial equilibrium when $\varphi>1$ or when $\xi<\max\{\underline{\xi},0\}$.

To sum up,

Lemma 3. *Full dispersion* ($\lambda=1/2$) *is a stable spatial configuration*

(i) *when* $\xi\geq\overline{\xi}$ *regardless of commuting costs* t;

(ii) *when* $\overline{\xi}>\xi>\varphi$ *if and only if* $t>\overline{t}_2$;

(iii) *when* $\xi=\varphi$ *if and only if* $t>\overline{t}_3$;

(iv) *when* $\xi < \varphi$ *if and only if* $\bar{t}_1 > t > \bar{t}_2$ *and* $\varphi > \xi > \max\{\underline{\xi}, 0\}$.

Stability for a partial agglomeration equilibrium: In a partial agglomeration equilibrium $\lambda^* \in (1/2, 1)$ such that $\Gamma(\lambda^*, t) = 0$ is stable if and only if $\mathrm{d}\Gamma/\mathrm{d}\lambda < 0$ when $\lambda = \lambda^*$. Note that $\Gamma(\lambda^*, t) = 0$ has at most one solution when $\lambda \in (1/2, 1)$. Let λ^i be the implicit solution of $\Gamma(\lambda, t) = 0$. If $\Gamma(1/2, t) < 0$ (where $\Gamma(1/2, t) = \mathrm{d}\Delta\mathbf{EV}/\mathrm{d}\lambda$ when $\lambda^* = 1/2$) and $\Delta\mathbf{EV}(1) > 0$, then λ^* exists but is unstable. By contrast, if $\Gamma(1/2, t) > 0$ and $\Delta\mathbf{EV}(1) < 0$, then λ^* exists and is stable. In other words, an *asymmetric spatial configuration* emerges when commuting costs take intermediate values. In addition, under this spatial configuration, we have

$$L_A^* - L_B^* = \left(\lambda^* - \frac{1}{2}\right)\Lambda(\lambda^*)$$

where

$$\Lambda \equiv \frac{2(1-\xi+\varphi) + t(\varphi-\xi)(1-\lambda^*)\lambda^*}{(1+t\lambda^*/2)[1+t(1-\lambda^*)/2]}$$

and where $L_A^* = L_B^*$ when $\lambda^* = 1/2$ and $L_A^* > L_B^*$ when $\lambda^* > 1/2$ because $\partial\Lambda(\lambda)/\partial\lambda > 0$.□

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