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Dual Labor Markets, Urban Unemployment, and Multicentric Cities*

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A two-sector model of urban employment is developed which focuses on the formation of a secondary sector in response to a primary-sector demand shock. The optimal location of this (single-firm) sector is shown to give rise to a multicentric urban spatial structure. Conditions are then established under which the new labor market equilibrium involves both a decrease in unemployment and an increase in net income for those unemployed. These results are extended to a case where all unemployment benefits are financed by local taxation of firms. Here it is shown that profit incentives may exist for the primary sector to subsidize entry of the secondary sector. *Journal of Economic Literature* Classification Numbers: J41, R14.

1. INTRODUCTION

In this paper a two-sector model of urban labor markets is developed within an explicit spatial setting. This model extends the one-sector model of spatial urban labor markets developed in [38]. The central focus of the present model is on the formation of a secondary sector under conditions

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in which a demand shock in the primary sector leads to a sharp increase in urban unemployment. The entry of this sector (here treated as a single firm) gives rise to a distinct secondary employment center-thus establishing a link between two major themes in the economic literature: "labor dualism" and "multicentric cities". It is the synthesis of these two theoretical perspectives which motivates our present approach.

Turning first to "labor dualism", this concept can be characterized formally in terms of a labor market with two distinct employment sectors: "primary" and "secondary" [8]. In the primary sector, firms offer stable employment, relatively high wages, and a stimulating work environment. In the secondary sector these features are absent. Jobs tend to be short-term, low-wage, and menial in nature. The wage differences between sectors can in part be viewed as reflecting the presence of a "efficiency wage" in the primary sector, which is above the market-clearing wage in the secondary sector [1]. In particular, when the relative complexity of work tasks in the primary sector make it more difficult to monitor worker performance, it has been argued that such a wage premium serves to discourage shirking, and hence to maintain output quality [33]. In addition, this argument has been used to explain how equally skilled workers can earn different wage levels in equilibrium. Hence in the present model we adopt the view that these two sectors are differentiated by the relative difficulty of monitoring worker performance.[1]

However, these distinctions fail to capture one of the most basic features of labor dualism, namely its relation to unemployment. Indeed, the very existence of secondary-sector activities may depend on the availability of a sufficiently large unemployment pool to render them economically viable. Hence such activities may only arise during periods of reduced demand in the primary sector. It has thus been argued by many that the formation of a secondary sector is fundamentally an endogenous phenomenon, which is largely contingent on the occurrence of demand shocks [25, 30, 32]. This serves to motivate the "demand-shock" scenario which plays a major role in the present model.

Turning next to concept of "multicentric cities", we begin by noting that the vast majority of spatial urban economic models have thus far focused exclusively on the concept of a "monocentric city" in which all employment is concentrated in the CBD (as developed for example in [11]). However, it has long been recognized that secondary employment centers (SEC's) are

[1] Other approaches are possible here. In "turnover" models, for example, it is assumed that primary sector workers have higher job mobility. Alternatively, wage differentials can arise from external factors such as stronger labor unions in primary sectors. However, in the present model we focus on differences among job types rather than among worker skills or other external factors. This approach appears to be supported by empirical studies (e.g., [7, 22]) and has become standard in dual labor market theory (e.g., [2, 5, 21, 22, 28]).

an important part of the urban landscape (as documented for example in [9, 13, 24, 26, 27]). Hence a number of theoretical models have been proposed to account for this locational behavior by firms (for example, [12, 17, 18, 31, 37]). In particular, [12] models the location behavior of a single large firm entering a city where all existing firms are concentrated in the CBD. Assuming that the labor demand of this firm is a significant portion of total labor supply, any location choice outside the CBD can be regarded as the formation of a secondary employment center.

But the labor market assumptions embodied in these multicentric city models have thus far been quite restrictive. In particular, none of these models consider the possibility of unemployment. The only possible job creation in SEC's is in terms of new migrants to the city. More generally, such "open city" models implicitly assume that jobs are always available somewhere in the system, and hence that in equilibrium all workers are guaranteed the same *exogenously determined* level of utility.

Hence the present paper attempts to synthesize these two lines of development in a manner which overcomes some of their present limitations. This effort was begun in [38], where an urban labor market with involuntary unemployment was modeled in the context of a monocentric city. The labor market model was based on the shirking version of the efficiency wage theory (mentioned above) in which imperfect monitoring of workers leads to an equilibrium wage premium just sufficient to discourage shirking. This efficiency wage exceeds the market-clearing wage, and results in an excess supply of labor. Here transitions in employment status are modeled as a Markov process in which unemployed workers have some incentive to wait for jobs. Hence the steady state of this process involves a permanent level of involuntary unemployment. In this context, it is shown that if employed and unemployed workers differ with respect to incomes but not in land consumption, then employed workers will always outbid the unemployed for residential locations nearest the CBD (containing all firms).[2] Moving is assumed to be costless, so that in the steady state there will be a continual flow of intraurban movers between a central "employment zone" and a peripheral "unemployment zone".[3]

This model is extended in the present paper by allowing for the possible entry of a secondary sector during periods of demand shock in the primary sector. To permit a tractable analysis of spatial impacts, we follow the above mentioned approach of [12] by taking the primary sector to consist of a large number of small firms concentrated in the CBD, and treating the

[2] However, if employed and unemployed workers are allowed to differ in their land consumption, then this spatial pattern can be reversed [14].

[3] Alternatively, one may consider cases in which workers who become unemployed maintain their locations, but reduce their consumption of other goods [10].

secondary sector as a single large firm (say a large assembly-line production plant with low-skill jobs). In this spatial context, we address the following questions: Under what conditions will the secondary sector form? Where will it locate? What will be its impact on unemployment? When will the government have an incentive to subsidize its entry? To answer these questions, we model the demand-shock scenario as three-stage process. The first stage corresponds essentially to the model in [38], where strong demand for the primary good leaves a steady-state pool of unemployed workers which is too small to warrant entry of the secondary sector. The demand shock occurs in the second stage, leading to both an increase in unemployment and a decrease in the efficiency wage level (resulting in part from the "worker discipline" effect of unemployment observed in [33]). These effects together allow profitable entry of the secondary-sector firm in the third stage. The optimal location of this firm is always near the edge of the city (where the unemployed reside), so that entry is synonymous with the formation of a secondary employment center. In addition, the lack of monitoring required for its low-skill jobs removes any need for a wage premium in this sector. However, there is assumed to exist a mandated minimum-wage level which exceeds the "welfare wage" received by the unemployed. Hence there now exist wage differentials (and corresponding utility differentials) between all three employment categories: primary, secondary, and unemployed. As an extension of the Markov model in [38], these differentials are maintained by a three-state employment transition process in which secondary-sector employment is always preferable to unemployment but not to primary-sector employment. In this context, the main result of the paper is to establish conditions under which the entry of a secondary sector leads to a new steady state in which there is not only in a decrease of unemployment, but also in an increase in the net income of those unemployed.

It may be argued, however, that this model is overly optimistic in that it assumes all unemployment benefits are financed exogenously by a federal government. Hence in the second half of the paper this assumption is relaxed by requiring that all unemployment benefits be financed by local taxation of firms. In this context, it is shown that when taxation discourages entry of a secondary sector, there may be profit incentives for the primary sector to subsidize the entry of a secondary sector, and in particular to pay all taxes for this sector.[4] Conditions are established under which the resulting two-sector equilibrium involves not only increased profits for the primary sector, but also increased employment levels and net income for the

[4] For a tax/subsidy analysis of a nonspatial dual labor market with efficiency wages, see for example [19, 23] and [28, Chapter 8].

unemployed. This aspect of the model is often observed in cases where local governments use tax subsidies to attract new firms to their regions.[5]

To develop these results, we begin in the next section with a brief description of the basic one-sector model, and refer the reader to [38] for further elaboration. In addition, certain technical aspects of the following development have been omitted, and can be found in [34].

2. THE ONE-SECTOR MODEL

In this section, we briefly describe the one-sector model, and refer the reader to the development in [38] for further elaboration. Basically, this model is taken to describe the primary sector of the city prior to the entry of the secondary sector. As in [38] we postulate a closed monocentric city model with absentee land ownership in which residential locations are uniformly distributed with unit density along a one-dimensional continuum from the CBD. Their are N identical households, each consuming one unit of land outside the CBD, and providing one unit of labor (worker). The primary production sector consists of M identical firms, each located in the CBD and consuming no land. The monitoring technology of firms is imperfect, and each uses wage incentives to discourage workers from shirking. But wage levels high enough to discourage shirking also lead to an excess supply of labor (involuntary unemployment). The employed are differentiated from the unemployed not only by wages, but also their commuting costs. Employed workers incur an annual commuting cost of τ dollars per unit of distance, and in addition, take α shopping (and other nonwork) trips for every commuting trip, resulting in a total annual *travel cost* of $(1+\alpha)\tau$ per unit of distance from the CBD. Unemployed workers incur only shopping costs of $\alpha\tau$ per unit of distance.

In this framework, it is assumed that transitions between employment and unemployment for any given worker are governed by a *two-state* (*time homogeneous*) *Markov process*. Firms are then assumed to seek the minimum wage level which will ensure that the expected life-time utility of a non-shirking worker is not lower than that of a shirker.[6] In [38] it is shown that the steady-state *efficiency wage level*, $w(L_1)$, required to achieve

[5] For example, [20] and [36] show that favorable local taxation policies are a major factor in decisions by large overseas firms to locate within Europe. A dramatic case in point has been the emergence of "Silicon Glen" in Scotland (see for example [15, 16]). There are numerous examples of this in the U.S. as well, including the location of Toyota in Kentucky, and Volkswagen in Pennsylvania.

[6] As in [38], all utilities are hypothesized to be expressible in terms of *net income*, and will be made more explicit in Section 4.3 below.

an employment level of L_1 in the primary sector (denoted as sector 1) is given by[7]

$$w(L_1) = (w_0 + e_1) + \tau L_1 + S(L_1), \tag{2.1}$$

where w_0 is the fixed government *welfare payment* to the unemployed,[8] e_1 is the *effort expenditure* required by workers for full productivity in the primary sector, and where the *wage surplus* term

$$S(L_1) = \frac{e_1}{x(L_1)}\left(\frac{sN}{N - L_1} + \sigma\right). \tag{2.2}$$

represents the portion of wages directly attributable to the prevention of shirking behavior (as discussed in [38]). Here $x(L_1)$ is the *detection rate* (which is assumed to be a decreasing function of the average size of firms, L_1/M), s is the (exogenous) job *separation rate*, and σ is the *discount rate*. The only new element in the present development is the introduction of a *minimum wage level*, $\underline{w}$, which firms must pay. Hence the relevant *effective wage* at each employment level L_1 is given by

$$w_1(L_1) = \max\{\underline{w}, w(L_1)\}. \tag{2.3}$$

If the minimum wage is greater that the welfare payment plus the basic effort expenditure by workers, i.e., if $\underline{w} > w_0 + e_1$, then this constraint will be binding for sufficiently small employment levels. In particular, the employment level, $\underline{L}$, given by $w_1(\underline{L}) = \underline{w}$, is seen to define the range of employment levels in which effective wages are efficiency wages, now designated as the *efficiency-wage range*, $[\underline{L}, N]$.

The profit-maximizing model of primary-sector firms is identical to that in [38] except that firms are constrained by the minimum-wage requirement, and hence must now pay the *effective* wage in (2.3) to guarantee non-shirking behavior by workers. Hence, given market price, p_1, the decision problem for each firm, j, is to choose that combination of wage level, w_j, and employment level, l_j, which maximizes its profits

$$\pi_1(l_j, w_j) = p_1 f_1(l_j) - w_j l_j, \tag{2.4}$$

subject to the conditions that $w_j \geqslant w_1(L_1)$ and $l_j \geqslant 0$. If the *aggregate production function* for sector 1 is given by $F_1(L_1) = Mf_1(L_1/M)$, the

[7] This is the same as expression (2.21) in [38], except that τ in [38] was assumed to be the *total* annual travel cost per unit of distance, and $(1 - \alpha)\tau$ was the fraction attributable to commuting.

[8] This external-funding assumption is relaxed in Section 5 below.

equilibrium profit maximization condition in (2.30) of [38] now has the form

$$p_1 F_1'(L_1) = w_1(L_1). \tag{2.5}$$

By a straightforward modification of Theorem 2.1 in [38] it can be shown that if firms in sector 1 are *productive* in the sense that positive profits can be earned for the first worker hired, i.e., if

$$F_1'(0) > \max\{\underline{w}, w_0 + e_1\}/p_1, \tag{2.6}$$

then *there always exists a unique one-sector equilibrium*, L_1, and associated wage level, $w_1 = w_1(L_1)$, satisfying (2.1) and (2.5) (where we are of course primarily interested in those equilibria with employment levels in the *efficiency wage range*, $[\underline{L}, N]$). Finally we note that the equilibrium spatial configuration of households is here identical to that in [38], with employed workers living closest to the CBD and unemployed workers occupying the periphery of the (monocentric) city. This spatial equilibrium will be elaborated further in Section 4.3 below.

3. THE TWO-SECTOR MODEL

The basic hypothesis of the two-sector (or "dual sector") model is that the secondary sector only forms when there is a sufficiently large unemployment pool to allow positive profits to be earned. In the present case, the secondary sector is assumed to operate on a low profit margin per worker, so that the fixed costs of setting up production make it unprofitable to operate at low levels of employment. Hence this sector, now designated as *sector* 2, is treated as a single firm which requires a large work force of relatively unskilled labor. For simplicity it is assumed that output in sector 2 is entirely unrelated to that in sector 1, and hence that the two sectors interact only in terms of their labor inputs. We begin by developing an explicit model of sector 2, and then analyze the consequences of this new sector for job turnovers and efficiency wages.

3.1. *A Model of Production in the Secondary Sector*

As in sector 1, production technology for sector 2 is taken to be representable by a *production function*, $F_2(L_2)$, which is assumed to be twice differentiable with $F_2(0) = 0$, $F_2'(L_2) > 0$ and $F_2''(L_2) \leqslant 0$ for all L. Similarly, it is assumed that all output can be sold at a fixed *market price*, p_2. Unlike sector 1, however, production tasks in sector 2 are taken to be relatively simple and easily monitored. Hence it is assumed at the effort expenditure required for production is effectively zero, and that shirking behavior can be ignored.

A more important difference between these sectors relates to the production decision itself. In particular, any production decision by sector 2 must necessarily involve a decision to *locate* within the city. Here several factors are important. First, in deciding whether production is profitable at all, the firm must take into account not only labor costs, but also the *fixed costs*, c_2, of setting up operations in the city. Moreover, in choosing a specific location (distance from CBD), d, the firm is assumed to take into account both *location rent*, $R(d)$, and the accessibility of d to the CBD, where the latter effect is represented by an annual CBD-*interaction cost*, ε, per unit of distance (reflecting the occasional need to utilize city services in the CBD and/or interact with primary sector firms). Both of these location-specific costs are taken to be *small* relative to fixed costs c_2, and serve primarily to reflect the fact that *other things being equal* the firm is assumed to prefer locations with lower rents and closer to the CBD. Under these assumptions, the *profit function* for sector 2 is given for each labor-wage-location combination, (L_2, w_2, d_2), by[9]

$$\pi_2(L_2, w_2, d_2) = p_2 F_2(L_2) - w_2 L_2 - R(d_2) - \varepsilon d_2 - c_2. \tag{3.1}$$

Second, the labor supplied to sector 2 will depend not only on the wage offer, w_2, but also on the location, d_2, chosen by the firm. Hence, if the appropriate *labor supply function* for sector 2 is denoted by $N_2(w_2, d_2)$, then (recalling that all firms are required to pay at least the minimum wage, $\underline{w}$) the relevant *profit-maximization problem* for sector 2 now takes the form

$$\max_{L_2, w_2, d_2} \pi_2(L_2, w_2, d_2), \quad \text{subject to} \quad L_2 \geqslant N_2(w_2, d_2), \quad w_2 \geqslant \underline{w}, \quad d_2 \geqslant 0. \tag{3.2}$$

For our later purposes, the most important instances of this problem are those in which there exist locations in the city where sector 2 can locate, offer the minimum wage, and attract enough workers to earn maximum (positive) profits at this wage level. By our assumptions on F_2, this profit-maximizing employment level, $\bar{L}_2$, at the minimum wage, $\underline{w}$, is uniquely determined by the first-order condition,

$$p_2 F_2'(\bar{L}_2) = \underline{w}. \tag{3.3}$$

[9] Note that while land consumption is implicitly taken to be unity, the actual extent of land area occupied by the firm is ignored and, as with sector 1 firms, is formally treated as a *point* location Hence the primary role of the rent term is allow "rent effects" to be incorporated into the location decision in a simple way. The role of interaction costs, ε, is similar. See footnote 13 below for additional discussion.

Moreover, since this optimal employment level is *independent* of location, it follows that whenever there is at least one location where $\bar{L}_2$ workers can be hired at the minimum wage, then the firm has no incentive to offer a higher wage. Hence in this case, the profit-maximization problem for sector 2 reduces to finding that location, d_2, with $N_2(\underline{w}, d_2) \geqslant \bar{L}_2$, which minimizes rent plus CBD-interaction cost.

3.2. *A Two-Sector Model of Job Turnover*

Given this model of production in sector 2, we now consider the labor market for two sectors. In particular, we assume that sectors 1 and 2 are both present in the city, and that the employment status for individuals is governed by a *three-state* time-homogeneous Markov process with states, $\{0, 1, 2\}$, corresponding respectively to "unemployed", "employed in sector 1", and "employed in sector 2". Much of the following development closely parallels that for the two-state Markov process in [38]. Hence we focus mainly on the new elements involved in the three-state case. The present process is again completely defined by the *transition probabilities*, $P_t(i, j)$, of being in employment state j at time t given state i at time zero. If $T_i^{(2)}$ denotes the duration time in state i for this two-sector model, then as in expression (2.10) of [38], it follows that $T_i^{(2)}$ is exponentially distributed, i.e. that

$$P(T_i^{(2)} > t) = \exp(-\lambda_i^{(2)} t), \qquad t \geqslant 0, \tag{3.4}$$

where $\lambda_i^{(2)}$ represents the exit rate from state $i = 0, 1, 2$. But unlike the two-state model, transition probabilities for the three-state model cannot be parametrized entirely in terms of these exit rates. We also require the "*jump*" *probabilities*, p_{ij}, that an individual leaving state i will next occupy state j. The desired Markov process is then completely definable in terms of (destination-specific) *exit rates*, $\lambda_{ij}^{(2)} = \lambda_i^{(2)} p_{ij}$, from state i to state j, for all distinct states $i, j = 0, 1, 2$ (see [6, Section 8.4]). To determine these rates, we must impose several additional behavioral assumptions. First we extend the basic premise of the one-sector model that "employment" is preferable to "unemployment". In particular, if the spatial equilibrium utility levels of nonworkers and workers in sectors 1 and 2 are denoted respectively by U_0, U_1, and U_2, then we now assume that[10]

$$U_1 > U_2 > U_0. \tag{3.5}$$

To derive the desired exit rates, we next postulate that (as in [38]) all transitions are governed by independent "job-offer" events and "job-separation" events, where for the present it is assumed that *all workers are*

[10] Conditions under which (3.5) holds for these utilities will be developed in Section 4.3.

nonshirkers. In particular, each individual is assumed to experience independent streams of separation events together with job offers from sectors 1 and 2. In addition, it is assumed that individuals react only to events which yield relevant changes in their present states. In particular, nonworkers always take job offers [by (3.5)], but effectively "ignore" separation events, which yield no relevant change of state. Similarly, workers always leave their jobs when experiencing separation events, but ignore job offers from the same sector (which are all assumed to be identical). Finally, [again by (3.5)] sector 2 workers will accept sector 1 job offers, but sector 1 workers will ignore sector 2 offers. If the (time homogeneous) *waiting times* to the next separation event and job offers in sectors 1 and 2 are denoted, respectively, by T_s, T_1, and T_2, then these assumptions can be formalized as follows. First observe that the duration time, $T_0^{(2)}$, of unemployment is precisely the time to the first job offer, so that by definition, $T_0^{(2)} = \min\{T_1, T_2\}$. Similarly, the duration time, $T_2^{(2)}$, for sector 2 is the waiting time to the first sector 1 job offer or separation event, so that again by definition, $T_2^{(2)} = \min\{T_1, T_s\}$. Finally, since workers only exit the primary sector through separation events, the duration time, $T_1^{(2)}$, is precisely the waiting time to separation, i.e., $T_1^{(2)} = T_s$. Given these behavioral assumptions, it is shown in [34] that the desired exit rates are given by

$$\lambda_{01}^{(2)} = \lambda_0^{(2)} p_{01} = a, \tag{3.6}$$

$$\lambda_{02}^{(2)} = \lambda_0^{(2)} p_{02} = b, \tag{3.7}$$

$$\lambda_{10}^{(2)} = \lambda_1^{(2)} p_{10} = s, \tag{3.8}$$

$$\lambda_{12}^{(2)} = 0, \tag{3.9}$$

$$\lambda_{20}^{(2)} = \lambda_2^{(2)} p_{20} = s, \tag{3.10}$$

$$\lambda_{21}^{(2)} = \lambda_2^{(2)} p_{21} = a, \tag{3.11}$$

From (3.6) and (3.11) we see that a is the common exit rate from both the unemployment pool and sector 2 into sector 1, and hence represents the *primary-sector job acquisition rate* for all individuals. Similarly, (3.8) and (3.10) show that s is the common exit rate from both sectors 1 and 2 into the unemployment pool, and hence represents the *job-separation rate* for all workers (under nonshirking). Finally, (3.7) shows that the new parameter, b, is the exit rate from the unemployment pool into sector 2, and hence represents the *secondary-sector job acquisition rate* for nonworkers a parallel to (2.11) and (2.12) in [38]. It is shown in [34] that for the nonshirking case (NS), the transition probabilities from sector 1 to each of the sectors, $\{0, 1, 2\}$, are now given respectively by

$$p_t^{NS}(1,0) = \frac{s}{a+b+s} - \left(\frac{s}{a+b+s}\right) \exp[-(a+b+s)\,t] \tag{3.12}$$

$$P_t^{NS}(1,1) = \frac{a}{a+s} + \left(\frac{s}{a+s}\right) \exp[-(a+s)\,t] \tag{3.13}$$

$$P_t^{NS}(1,2) = \frac{bs}{(a+s)(a+b+s)} - \left(\frac{s}{a+s}\right) \exp[-(a+s)\,t] + \left(\frac{s}{a+b+s}\right) \exp[-(a+b+s)\,t]. \tag{3.14}$$

Next recall that shirking behavior is still possible in the primary sector. Hence for the shirking case (S) the exit rate, s, from the primary sector must again be augmented by a *detection rate*, x (as in the one-sector case). However, since the separation rate, s, is still valid for the secondary sector, it should be clear that the appropriate transition probabilities in this case cannot be obtained by simply replacing s everywhere by $s+x$. As shown in [34], the actual transition probabilities for the shirking case (S) are given by[11]

$$P_t^S(1,0) = \frac{(s+x)(a+s)}{(a+s+x)(a+b+s)} + \left(\frac{x(s+x)}{(a+s+x)(b-x)}\right) \exp[-(a+s+x)\,t] - \left(\frac{b(s+x)}{(a+b+s)(b-x)}\right) \exp[-(a+b+s)\,t] \tag{3.15}$$

$$P_t^S(1,1) = \frac{a}{a+s+x} + \left(\frac{s+x}{a+s+x}\right) \exp[-(a+s+x)\,t] \tag{3.16}$$

$$P_t^S(1,2) = \frac{b(s+x)}{(a+s+x)(a+b+s)} - \left(\frac{b(s+x)}{(a+s+x)(b-x)}\right) \exp[-(a+s+x)\,t] + \left(\frac{b(s+x)}{(a+b+s)(b-x)}\right) \exp[-(a+b+s)\,t]. \tag{3.17}$$

[11] It is important to observe that, unlike the *NS*-case above, these expressions for transition probabilities do *not* always hold. In particular, when the sector 2 job-acquisition rate, b, is the same as the separation rate, x, expressions (3.15) and (3.17) both involve division by zero. Moreover since both b and x are *system variables* which can quite possibly be equal, this singularity problem cannot be avoided. However, it is shown in [34] that while these transition probabilities have no simple closed-form expression when $b = x$, they are still well defined, and yield a closed-form expression for efficiency wages.

3.3. *Efficiency Wages for the Two-Sector Model*

Recall from the one-sector model of [38] that the expected lifetime utilities of workers depend on a given set of prevailing spatial equilibrium utility levels in all sectors. Hence, as in (3.5) above, we simply take such utilities to be given and derive a general expression for efficiency wages [a more explicit version will be developed in the next section below]. Let U_0, U_1, and U_2 denote a given set of spatial equilibrium utility levels for non-workers and workers in sectors 1 and 2, respectively, under the condition of nonshirking behavior. Note also that since shirkers are assumed to expend no effort, the (net income) utility level for a shirker in sector 1 is simply $U_1 + e_1$ (as in (2.2) of [38]). Given these utilities together with the discount rate, σ, in (2.2), the transition probabilities in (3.12) through (3.14) yield the following expression for the *expected lifetime utility*, V_{NS}, of a *nonshirker* sector 1, where $D = a + b + s + \sigma$,

$$V_{\mathrm{NS}} = \int_0^\infty [P_t^{\mathrm{NS}}(1,0)\, U_0 + P_t^{\mathrm{NS}}(1,1)\, U_1 + P_t^{\mathrm{NS}}(1,2)\, U_2] \exp(-\sigma t)\, dt$$

$$= \left(\frac{s}{\sigma D}\right) U_0 + \left(\frac{a+\sigma}{\sigma(a+s+\sigma)}\right) U_1 + \left(\frac{bs}{\sigma(a+s+\sigma)\, D}\right) U_2. \tag{3.18}$$

Similarly, the transition probabilities in (3.15) through (3.17) yield the following expression for the *expected lifetime utility*, V_{S}, of a *shirker* sector 1, where $G = a + s + x + \sigma$,

$$V_{\mathrm{S}} = \int_0^\infty [P_t^{\mathrm{S}}(1,0)\, U_0 + P_t^{\mathrm{S}}(1,1)\, U_1 + P_t^{\mathrm{S}}(1,2)\, U_2] \exp(-\sigma t)\, dt$$

$$= \left(\frac{(s+x)(a+s+\sigma)}{\sigma DG}\right) U_0 + \left(\frac{a+\sigma}{\sigma G}\right)(U_1 + e_1) + \left(\frac{b(s+x)}{\sigma DG}\right) U_2. \tag{3.19}$$

Next the utility (net income) for nonshirkers in sector 1 can be written as $U_1 = w_1 - C_1$, where w_1 is the *wage* in sector 1 and where C_1 denotes all relevant *costs* in sector 1 (including the effort expenditure, e_1). Hence (in a manner paralleling the argument in (2.19) through (2.21) in [38]), it follows that if we substitute this expression for U_1 and equate (3.18) and (3.19), then we may solve for the *efficiency wage*, w_1, in sector 1 to obtain

$$w_1 = \left(\frac{a+s+\sigma}{a+b+s+\sigma}\right) U_0 + \left(\frac{b}{a+b+s+\sigma}\right) U_2 + \left(\frac{a+s+x+\sigma}{x}\right) e_1 + C_1. \tag{3.20}$$

Finally, the values of system variables a and b which will support employment levels, L_1 and L_2, as a steady state for population N are given by (see [34])

$$a(L_1) = s\,\frac{L_1}{N - L_1} \tag{3.21}$$

$$b(L_1, L_2) = s\,\frac{L_2 N}{(N - L_1)(N - L_1 - L_2)}. \tag{3.22}$$

4. DEMAND-SHOCK SCENARIO

Given these general components of the two-sector model, we are now ready to develop the specific case outlined in the Introduction. Recall that our interest focuses on situations where a demand shock in the primary sector leads to a sharp increase in unemployment, setting the stage for entry of the secondary sector. This scenario can be formalized in terms of the following three stages.

4.1. *Stage One*: *Effective Full Employment*

The first stage is essentially the one-sector model sketched in Section 2 above. In particular, it is assumed that unemployment is sufficiently low to preclude profitable entry by sector 2. To formalize this condition, we first observe that at any location, d_2, there is a minimum level of employment required to earn nonnegative profits. Moreover, since rent is never less than the opportunity rent, $\bar{R}$, and since $\varepsilon d_2 \geqslant 0$, it follows that $\pi_2(L_2, w_2, d_2) \leqslant p_2 F_2(L_2) - w_2 L_2 - \bar{R} - c_2$, and hence that entry requires an employment level at least equal to the *minimum employment level*, $L_{\min}$, defined by the condition

$$p_2 F_2(L_{\min}) - w L_{\min} - \bar{R} - c_2 = 0. \tag{4.1}$$

Moreover, assuming that the current wage in sector 1 is above the minimum wage, it follows that any primary-sector employment level, L_1, yielding an unemployment level less than or equal to $L_{\min}$, i.e. satisfying

$$N - L_1 \leqslant L_{\min} \tag{4.2}$$

is sufficiently large to preclude entry of sector 2. Hence stage 1 is taken to be characterized by an employment level, L_1, large enough to satisfy (4.2) [and thus to constitute an "exclusionary" employment level for sector 2].

4.2. *Stage Two*: *A Demand Shock*

Next suppose that there is a demand shock in sector 1, as reflected by a sharp fall in the market price to $\bar{p}_1 < p_1$. We shall assume that this drop in price still allows the primary sector to survive, but at a much lower ("lean and mean") level of equilibrium employment, $\bar{L}_1$, given [as in (2.5)] by

$$\bar{p}_1 F'_1(\bar{L}_1) = w_1(\bar{L}_1). \tag{4.3}$$

The corresponding rise in city unemployment, $N - \bar{L}_1$, then provides a cheap labor pool which sets the stage for entry of the secondary sector. Within this setting, our interest focuses on cases where sector 2 can locate and earn maximum profits without the need for direct competition with sector 1. Such cases can be formalized in terms of the following three additional conditions on the new employment level, $\bar{L}_1$, in sector 1,

$$\bar{p}_1 F_1(\bar{L}_1) - w_1(\bar{L}_1)\, \bar{L}_1 > 0 \tag{4.4}$$

$$w_1(\bar{L}_1) > \underline{w} \tag{4.5}$$

$$N - \bar{L}_1 > \bar{L}_2, \tag{4.6}$$

where $\bar{L}_2$ is again defined by (3.3). These conditions state, respectively, that profits in sector 1 are still positive, wages in sector 1 [as defined by (2.3) and (2.1)] are still above the minimum wage, and that unemployment is larger than the employment level desired by sector 2 at entry. It should be clear from the last two conditions that there may now be an incentive for the secondary sector to enter the city.

4.3. *Stage Three*: *Entry of the Secondary Sector*

Given $\bar{p}_1$ and $\bar{L}_1$, the entry of sector 2 and resulting spatial equilibrium can be decomposed into two steps. First we show that the entry decision (and in particular the optimal location choice) of sector 2 does not depend on the actions of sector 1. To see this intuitively, note that the direct effect of this entry will be to improve the prospects of individuals not working in sector 1. This will in turn reduce the threat of being fired from sector 1, and hence lead to an increase in the efficiency wage level. Thus, the ultimate effect on sector 1 (given $\bar{p}_1$) will be to further reduce the employment level, $\bar{L}_1$, which will not influence the entry decision of sector 2 (as shown below).

To formalize these observations, it is convenient to adopt general notation for the equilibrium employment levels, L_1 and L_2, and to assume (for consistency) that

$$0 < L_1 < N - L_2. \tag{4.7}$$

[We shall return to the specialized values below.] If d_2 denotes the location choice of the sector 2 firm, which we now designate as the *secondary employment center* (SEC), then under condition (4.7) [together with the assumption that exactly L_2 workers are hired] we can derive the *equilibrium rent*, $R(d_2)$, at the SEC as follows (see Fig. 1). First recall that all non-work activities are assumed to be concentrated in the CBD, so that like all other city residents those workers commuting to the SEC must still travel to the CBD for shopping. Hence the utility (net income) of a sector 2 worker at distance d from the CBD is given (in a manner paralleling (2.1) in [38]) by[12]

$$U_2(d) = \underline{w} - e_2 - \alpha\tau d - \tau\,|d_2 - d| - R(d) \tag{4.8}$$

so that for any utility level, U_2, the resulting bid rent function for sector 2 workers is of the form

$$R_2(d, U_2) = \max\{\underline{w} - e_2 - \alpha\tau d - \tau\,|d_2 - d| - U_2, 0\}. \tag{4.9}$$

Hence in equilibrium, the slope of the rent function in all locations, d, occupied by sector 2 workers is given by the slope, $R_2'(d, U_2)$, which is seen to be $(1-\alpha)\,\tau$ for all $d < d_2$, and $-(1+\alpha)\,\tau$ for all $d > d_2$. Moreover, since the locations occupied by sector 2 workers must form a connected interval containing d_2, it may readily be verified (see [34]) that if $\bar{d}_2 = N - L_2(1-\alpha)/2$, then the only possible equilibria for $d_2 > \bar{d}_2$ involve an isolated "suburb" of sector 2 workers surrounding the work site d_2. Similarly, if $\underline{d}_2 = N - (1/2)\,L_2$, then the only possible equilibria for $\underline{d}_2 \leqslant d_2 \leqslant \bar{d}_2$ involve an "edge city" of sector 2 workers. If $L_1 + (1/2)\,L_2 \leqslant d_2 < \underline{d}_2$ then the only possible equilibria are "subcenters" around d_2. Finally, since locations, $d_2 < L_1 + (1/2)\,L_2$, imply that sector 2 workers must compete for land with sector 1 workers in equilibrium, it follows that rents must be even higher in this range. Hence for each relevant location, d_2, the only possible *equilibrium rent*, $R(d_2)$, is given by

$$R(d_2) = \begin{cases} \bar{R} + \tau\left(\dfrac{1+\alpha}{2}\right) L_2 + \alpha\tau\left[N - \left(\dfrac{1}{2}\right) L_2 - d_2\right], & \\ \qquad\qquad\qquad\qquad\qquad \text{if } \ L_1 + \left(\dfrac{1}{2}\right) L_2 \leqslant d_2 < \underline{d}_2 & \\ \bar{R} + \tau(1+\alpha)(N - d_2), & \text{if } \ \underline{d}_2 \leqslant d_2 \leqslant \bar{d}_2 \\ \bar{R} + \tau(1-\alpha)\left(\dfrac{1+\alpha}{2}\right) L_2, & \text{if } \ d_2 > \bar{d}_2. \end{cases} \tag{4.10}$$

[12] Note also that (as with primary-sector workers) we assume work trips to be separate from shopping trips, so that those secondary-sector workers living beyond the SEC must still initiate shopping trips from their residences rather than from their work site.

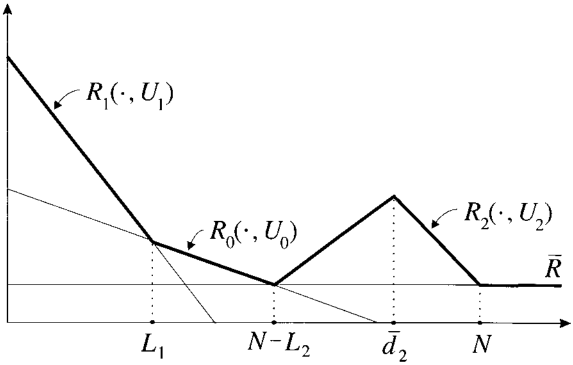

FIG. 1. Equilibrium rent for stage 3.

Note also that if the (positive) CBD-interaction costs, ε, are sufficiently small, and in particular, if

$$0 < \varepsilon < \alpha\tau, \tag{4.11}$$

then the unique location which minimizes the function, $R(d_2) + \varepsilon d_2$, is given by

$$\bar{d}_2 = N - \left(\frac{1-\alpha}{2}\right) L_2. \tag{4.12}$$

Hence, assuming that sector 2 can hire L_2 workers at $\bar{d}_2$ [i.e., that $N_2(\underline{w}, \bar{d}_2) \geqslant L_2$], it will follow from the argument at the end of Section 3.1 that $\bar{d}_2$ must be the unique equilibrium (profit-maximizing) location for sector 2.[13] We henceforth assume that positive profits can be earned at this location, i.e., that

$$p_2 F_2(L_2) - \underline{w} L_2 - [\bar{R} + \tau(1+\alpha)(N - \bar{d}_2)] - c_2 - \varepsilon \bar{d}_2 > 0. \tag{4.13}$$

Given this optimal location, we can then obtain the following expressions for the equilibrium utility levels, U_0, U_1, and U_2 (see [34]):

[13] As mentioned in footnote 9 above, the role of the term, $R(d_2) + \varepsilon d_2$, in the profit function, π_2, is to guarantee the existence of a unique profit-maximizing location for sector 2 which exhibits reasonable properties. An alternative approach (which avoids the need to specify either the land consumption or CBD-interaction cost of the firm) is to remove the term, $R(d_2) + \varepsilon d_2$, from π_2 and simply assume that the firm's preferences among locations are *lexicographically ordered* with respect to the pairs $[R(d_2), d_2]$, so that location d_2 is preferred to d_2' iff either $R(d_2) < R(d_2')$ or $[R(d_2) = R(d_2'), d_2 < d_2']$.

$$U_0 = w_0 - \alpha\tau(N - L_2) - \bar{R}, \tag{4.14}$$

$$U_1 = w_1 - e_1 - \alpha\tau(N - L_2) - \tau L_1 - \bar{R} \tag{4.15}$$

$$U_2 = \underline{w} - e_2 - \alpha\tau(N - L_2) - \tau\left(\frac{1+\alpha}{2}\right)L_2 - \bar{R}. \tag{4.16}$$

Note that U_1 involves the equilibrium wage, w_1, which has yet to be determined. But even without determining w_1, we can make a number of useful observations at this stage. To do so, it is useful to identify the equilibrium *space costs* (i.e., rent plus travel costs) for the three sectors, which are seen from (4.14), (4.15), and (4.16) to be given respectively by

$$SC_0 = \alpha\tau(N - L_2) + \bar{R} \tag{4.17}$$

$$SC_1 = \alpha\tau(N - L_2) + \tau L_1 + \bar{R} \tag{4.18}$$

$$SC_2 = \alpha\tau(N - L_2) + \tau\left(\frac{1+\alpha}{2}\right)L_2 + \bar{R} \tag{4.19}$$

yielding the following *space-cost differentials* for sectors 1 and 2 relative to nonworkers:

$$SC_1 - SC_0 = \tau L_1 \tag{4.20}$$

$$SC_2 - SC_0 = \tau\left(\frac{1+\alpha}{2}\right)L_2. \tag{4.21}$$

In these terms, we can express the equilibrium condition, $U_0 < U_2 < U_1$, in (3.5) a more explicitly form as follows. If in sectors 1 and 2 we now designate wages minus effort and space cost differentials as the *net wages* relative to nonworkers, then by eliminating common terms in (4.14), (4.16), and (4.15), condition (3.5) is seen to be equivalent to the following pair of net-wage conditions:

$$w_0 < \underline{w} - e_2 - \tau\left(\frac{1+\alpha}{2}\right)L_2 \tag{4.22}$$

$$\underline{w} - e_2 - \tau\left(\frac{1+\alpha}{2}\right)L_2 < w_1 - e_1 - \tau L_1. \tag{4.23}$$

The first condition (which we henceforth assume) requires that the minimum wage be sufficiently greater than the welfare payment to cover both the effort and space-cost differential incurred by sector 2 workers. The second condition requires simply that net wages in the primary sector be greater than in the secondary sector. This condition involves the efficiency wage level for sector 1, which is yet to be determined.

4.4. *Efficiency Wages*

To establish an explicit form for the efficiency wage in (3.20), we first observe from (4.15) that the value of C_1 in (3.20) is now given by

$$C_1 = w_1 - U_1 = e_1 + \alpha\tau(N - L_2) + \tau L_1 + \bar{R}. \tag{4.24}$$

Hence by substituting (3.21), (3.22), and (4.24) into (3.20) and reducing terms, we see that the *two-sector efficiency wage* can now be written as an explicit function of L_1 and L_2,

$$w(L_1, L_2) = \left\{ \theta(L_1, L_2)\, w_0 + [1 - \theta(L_1, L_2)] \left[\underline{w} - e_2 - \tau\left(\frac{1+\alpha}{2}\right) L_2 \right] \right\} + e_1 + \tau L_1 + S(L_1), \tag{4.25}$$

where $S(L_1)$ is again given by (2.2) and where the weighting factor, $\theta(L_1, L_2) \in (0, 1)$, is given by

$$\theta(L_1, L_2) = \frac{[\sigma(N - L_1) + sN](N - L_1 - L_2)}{sNL_2 + [\sigma(N - L_1) + sN](N - L_1 - L_2)}. \tag{4.26}$$

A comparison of (4.25) and (2.1) shows that this new efficiency wage has essentially the same form as in the one-sector case. In particular, the last three terms again correspond to the *effort cost*, *space-cost differential*, and *wage surplus* for sector 1. Hence all new features of the two-sector case are contained in the bracketed term in (4.25), which now plays the role of w_0 in the one-sector case. To interpret this term, we begin observing that in the one-sector case, w_0, essentially represented the *opportunity wage* which individuals could achieve outside the primary sector. In the present case, however, individuals outside the primary sector can expect to spend part of their time in sector 2 as well as sector 0. Hence the bracketed term in (4.25) [which is seen to be a convex combination of w_0 and the net wage, $\underline{w} - e_2 - \tau(1 + \alpha) L_2/2$, in sector 2] now represents the *composite opportunity wage* for individuals outside the primary sector.[14] Note finally, that since $\theta(L_1, 0) = 1$, the efficiency wage in the one-sector case is now formally the special case of (4.25) in which $L_2 = 0$. Moreover, since $L_2 > 0$ implies

[14] This convex-combination suggests that the composite wage might be the 'expected wage' for individuals moving between sectors 0 and 2. However the presence of σ in $\theta(L_1, L_2)$ shows that it is more accurately described as an 'expected discounted wage'.

$\theta(L_1, L_2) < 1$, it follows from (4.22) that the composite opportunity wage is strictly greater than w_0, and hence that

$$w(L_1, L_2) > w(L_1, 0) = w(L_1), \qquad L_1, L_2 > 0. \tag{4.27}$$

Additional properties of this efficiency wage function are detailed in [34].

4.5. *Labor Market Equilibrium*

Once sector 2 locates in the city at $\bar{d}_2$ and seeks to hire $\bar{L}_2$ workers at the minimum wage, $\underline{w}$, the situation changes for the primary sector. In particular, the relevant *effective wage* for this sector is now given by

$$w_1(L_1, \bar{L}_2) = \max\{\underline{w}, w(L_1, \bar{L}_2)\}, \tag{4.28}$$

where $w(L_1, \bar{L}_2)$ is the two-sector efficiency wage evaluated at $\bar{L}_2$ (as shown in Fig. 2, with new efficiency-wage range $[\underline{L}^\circ, N]$). Hence, as a parallel to (2.5), it follows from (4.28) [together with the given market price, $\bar{p}_1$] that the new employment level, L_1^*, in sector 1 must now be given by (see Fig. 2)

$$\bar{p}_1 F_1'(L_1^*) = w_1(L_1^*, \bar{L}_2). \tag{4.29}$$

To ensure that positive profits can be earned, it follows from the monotonicity properties of F_1' and w_1 that we must require $\bar{p}_1 F_1'(0) > w_1(0, \bar{L}_2)$. By employing (4.22) and (4.25) [with $L_2 = \bar{L}_2$], one easily sees that it is enough to require that the following parallel to condition (2.6) hold:

$$F_1'(0) > \max\left\{\underline{w}, \underline{w} + (e_1 - e_2) - \tau\left(\frac{1+\alpha}{2}\right)\bar{L}_2\right\}\Big/\bar{p}_1. \tag{4.30}$$

But since the monotonicity properties of F_1' and w_1 also imply that there is at most one solution to (4.29), it then follows that L_1^* is the only possible equilibrium employment level for sector 1. Note moreover from (4.27) [and (4.5)] that $\bar{p}_1 F_1'(\bar{L}_1) = w(\bar{L}_1) < w(\bar{L}_1, \bar{L}_2) \leqslant w_1(\bar{L}_1, \bar{L}_2)$, which together with the above monotonicity properties implies that

$$L_1^* < \bar{L}_1 \tag{4.31}$$

(as shown in Fig. 2, where the dashed curve again represents the one-sector efficiency wage function).[15] Hence by (4.6) we see that (4.7) still holds for

[15] If taken in isolation, this inequality might seem to suggest that the secondary sector 'steals' workers from the primary sector. However, it should be clear in the present model that this employment reduction results only indirectly from the influence of sector 2 employment on efficientcy wages, which in turn affects the optimal hiring levels in sector 1.

L_1^* and $\bar{L}_2$, and may conclude from (4.13) [with $L_2 = \bar{L}_2$] that $\bar{L}_2$ is still the only possible equilibrium employment level for sector 2. Thus, to guarantee that the employment level, L_1^*, constitutes an equilibrium for the labor market as a whole, it suffices to require that the net-wage conditions (4.22) and (4.23) hold for $(L_1, L_2) = (L_1^*, \bar{L}_2)$. For any given minimum wage, $\underline{w} > w_0 + e_2 + \tau(1+\alpha)\,\bar{L}_2/2$, this reduces to the single condition

$$\underline{w} - e_2 - \tau\left(\frac{1+\alpha}{2}\right)\bar{L}_2 < w_1(L_1^*, \bar{L}_2) - e_1 - \tau L_1^*. \qquad (4.32)$$

Hence, if there exists an employment level, L_1^*, satisfying (4.29) and (4.32), we now designate L_1^* as the *two-sector equilibrium* given $(\bar{p}_1, \underline{w}, \bar{L}_2)$.

It should be clear from Fig. 2 that unique two-sector equilibria will exist under appropropriate conditions. To motivate one such condition, we begin by recalling that a basic premise of the present scenario is that entrance of secondary sector serves to absorb some of the unemployment created by the demand shock in the primary sector, thus to improve the employment situation. But in view of (4.31), it is possible that in extreme cases the total employment may *decrease*, i.e., that $L_1^* + \bar{L}_2 < \bar{L}_1$. This of course can never happen if $\bar{L}_2 > \bar{L}_1$. But while sector 2 may be large relative to individual sector 1 firms, the present scenario focuses on cases

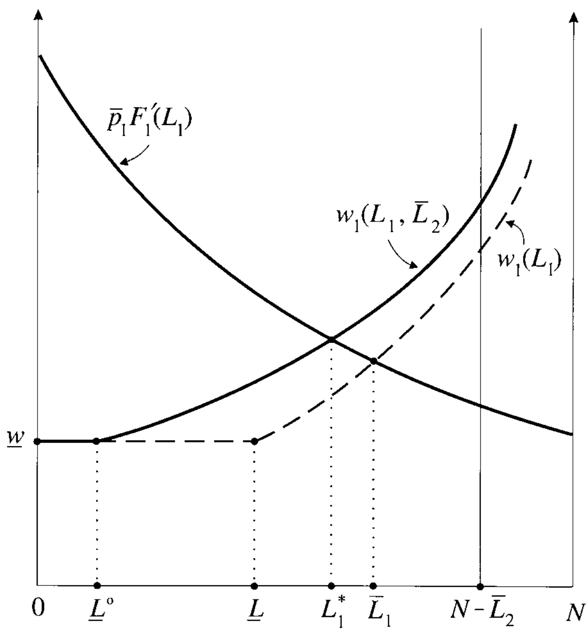

FIG. 2. Two-sector equilibrium.

where sector 2 is small in comparison to sector 1 as a whole. Hence, we interested in cases where the entrance of a relatively small sector 2 leads to equilibria with higher employment levels, $L_1^* + \bar{L}_2 > \bar{L}_1$, i.e., where $L_1^* > \bar{L}_1 - \bar{L}_2 > 0$. Our objective is to show that if the minimum wage is not too large, then this condition can be guaranteed. To establish the desired upper bound on admissible values of $\underline{w}$, we first observe from condition (4.3) and the monotonicity properties of F_1' and w_1 that

$$\bar{p}_1 F_1'(\bar{L}_1 - \bar{L}_2) > w_1(\bar{L}_1 - \bar{L}_2) \geqslant w(\bar{L}_1 - \bar{L}_2)$$
$$= w_0 + e_1 + \tau(\bar{L}_1 - \bar{L}_2) + S(\bar{L}_1 - \bar{L}_2) \tag{4.33}$$

and hence that the quantity

$$\bar{m} = \min\left\{\frac{S(\bar{L}_1 - \bar{L}_2)}{\theta(\bar{L}_1 - \bar{L}_2 - \bar{L}_2)}, \bar{p}_1 F_1'(\bar{L}_1 - \bar{L}_2)\right.$$
$$\left. - [w_0 + e_1 + \tau(\bar{L}_1 - \bar{L}_2) + S(\bar{L}_1 - \bar{L}_2)]\right\} \tag{4.34}$$

is always positive. Thus, if a minimum wage level, $\underline{w}$, is now said to be *admissible* iff

$$\left[w_0 + e_2 + \tau\left(\frac{1+\alpha}{2}\right)\bar{L}_2\right] < \underline{w} < \left[w_0 + e_2 + \tau\left(\frac{1+\alpha}{2}\right)\bar{L}_2\right] + \bar{m}, \tag{4.35}$$

then our main result is show that for such minimum wage levels, entry of sector 2 does indeed improve the unemployment situation both in sense that (i) unemployment is lower, and (ii) the utilities of the unemployed are higher. To state this result formally, let the equilibrium utilities for non-workers before and after the entry of sector 2 be denoted respectively by $\bar{U}_0$ and U_0^*. In these terms, if we now say that a two-sector equilibrium, L_1^*, *improves the unemployment situation* iff (i) $L_1^* + \bar{L}_2 > \bar{L}_1$, and (ii) $U_0^* > \bar{U}_0$, then it can be shown that (see [34]):

THEOREM 4.1. *For each admissible minimum wage level, $\underline{w}$, there exists a unique two-sector equilibrium, L_1^*, given $(\bar{p}_1, \underline{w}, \bar{L}_2)$. In addition, each such equilibrium improves the unemployment situation.*

5. TAX/SUBSIDY ANALYSIS

The above analysis assumes that all welfare payments are distributed by the Federal Government, and that no Federal income taxes are paid. To add realism to this "manna from heaven" scenario, we now introduce endogenous taxation. For sake of simplicity, however, we consider only the extreme case in which all city welfare payments (and municipal taxes) are financed by employee taxes on local firms. In particular, for any given employment level, L_1, the employee *welfare tax*, $t(L_1)$, must satisfy the condition that $t(L_1)\, L_1 = w_0(N - L_1)$, and hence is given by

$$t(L_1) = \max\left\{0,\, w_0\, \frac{N - L_1}{L_1}\right\}. \tag{5.1}$$

In addition, there is assumed to be a flat employee *municipal tax*, $t_0 > 0$, to cover the use of infrastructure and municipal services by firms. In this new setting, it follows that in the one-sector case the profit function for each firm, j, is now of the form

$$\pi_1(l_j, w_j) = p_1\, f_1(l_j) - [\, w_j + t(L_1) + t_0 \,]\, l_j, \tag{5.2}$$

where as before the total employment level, $L_1 (= \sum_{j=1}^{M} l_j)$, is treated as exogenous by each firm j. Given this modification, the analysis of the one-sector model in Section 2 can once again be carried out. In particular, the new *one-sector equilibrium* condition [paralleling (2.5)] is now given by

$$p_1 F_1'(L_1) = w_1(L_1) + t(L_1) + t_0, \tag{5.3}$$

where the effective wage, $w_1(L_1)$, is again given by (2.3) and (2.1). This new equilibrium situation is depicted in Fig. 3. Notice in particular from (5.1) that the right-hand side of (5.3) approaches infinity at the origin (as the number of unemployed per worker grows without bound). Hence in this endogenous taxation case there will generally be *multiple equilibria*, such as L_1^a and L_1^b in Fig. 3. However, it should also be clear from the monotonicity properties of F_1' that the equilibrium with highest employment level (such as L_1^b in Fig. 3) must yield the lowest value of marginal costs, $w_1(L_1) + t(L_1) + t_0$, and hence yield the unique *maximum-profit equilibrium* for firms. Moreover, in cases where there are only two equilibria (i.e., where F_1' is sufficiently smooth to rule out larger numbers of intersection points) it is easily verified that this maximum-profit equilibrium is the unique *stable* equilibrium point (see [34]). Hence we take this point to define the *one-sector equilibrium* (whenever it exists).

In this setting, our interest focuses on the consequences of such taxation for the "demand-shock scenario" in Section 4 above. If we now refer to the original model as the *no-tax case*, then it should be clear from the above observations that existence of equilibria is much more problematic in the present *tax case*. In particular, there will generally be a significant range of price levels (such as the price, $\hat{p}_1$, in Fig. 3) where a one-sector equilibrium exists for the no-tax case, but fails to exist for the tax case (which is hardly surprising in view of the additional costs borne by firms). Hence, rather than attempting to establish general conditions for existence of equilibria in this more complex setting, we shall be content to assume that the demand shock is not "too severe", and shall analyze the consequences for this case.

5.1. *Stages One and Two of the Demand-Shock Scenario*

Both the first and second stages of the demand-shock scenario for the tax case can be illustrated by Fig. 4. Stage 1 is characterized by a market price, p_1°, yielding a one-sector equilibrium, L_1°, with a relatively small level of unemployment satisfying (4.2). This ensures that the secondary sector is precluded from entry, even without considering taxes. For sake of comparison, stage 2 is again assumed to involve the *same* fall in market price to $\bar{p}_1$, which we now assume to be mild enough to allow primary sector firms to survive in the tax case (as in Fig. 4). This in turn yields a new equilibrium employment level, $\tilde{L}_1$, which is again assumed to satisfy the inequality, $w(\tilde{L}_1) > \underline{w}$, with respect to the prevailing minimum wage, $\underline{w}$, and hence to satisfy

$$\bar{p}_1 F'_1(\tilde{L}_1) = w(\tilde{L}_1) + t(\tilde{L}_1) + t_0. \tag{5.4}$$

In addition, $\tilde{L}_1$ is again assumed to be low enough to allow for possible entry of the secondary sector, i.e., to satisfy

$$N - \tilde{L}_1 > \bar{L}_2 \tag{5.5}$$

where $\bar{L}_2$ is again the profit-maximizing employment level for sector 2 given $\underline{w}$. In the present case, however, taxation creates an additional barrier to entry. Moreover, the relevant tax level for sector 2 cannot even be estimated without knowledge of the equilibrium unemployment level. Hence, unlike the simple no-tax case, the secondary sector must now be able to predict the behavior of the primary sector in order to determine its profits. To avoid this complication (which at a minimum requires a sophisticated calculation by the sector 2 firm) we make the simplifying assumption that even without considering the welfare tax, the flat-rate municipal tax, t_0, is sufficiently large to prohibit entry of sector 2. In terms of (4.13), this amounts to assuming that

$$\max_{L_2}\{p_2 F_2(L_2) - (\underline{w} + t_0)\, L_2\} < [\bar{R} + \tau(1+\alpha)(N - \bar{d}_2)] + c_2 + \varepsilon \bar{d}_2. \tag{5.6}$$

Hence if the secondary sector is required to pay taxes, then no third stage is possible. In this context, our objective is to show that it can be advantageous for the primary sector to *subsidize* entry of the secondary sector by assuming its tax burden.

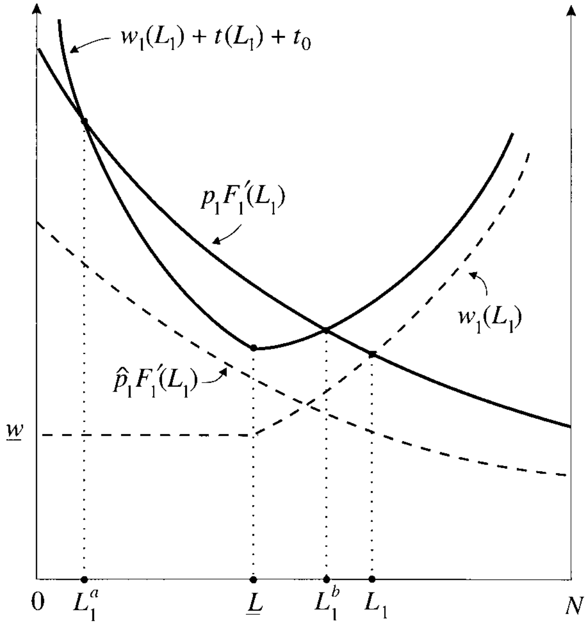

FIG. 3. One-sector equilibrium with taxation.

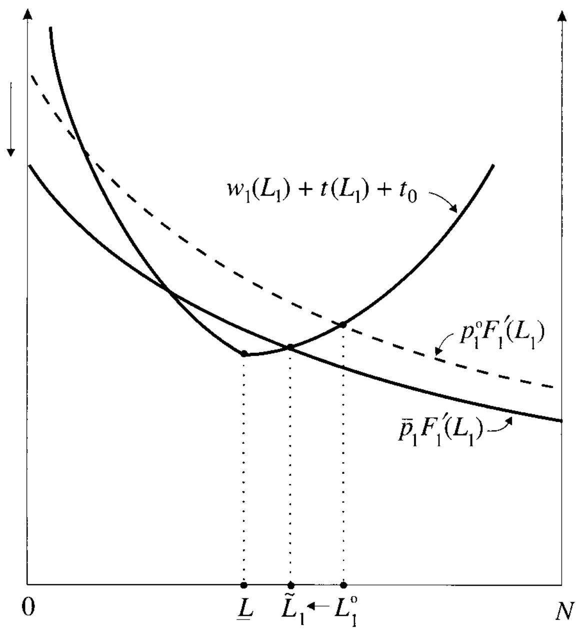

FIG. 4. Employment levels for stages 1 and 2.

5.2. *A Possible Stage Three for the Demand-Shock Scenario*

To do so, we now suppose that local government considers waving all taxes for sector 2. Assuming that tax revenues are still required, this is only possible if sector 1 pays the taxes for sector 2. While this would appear to be unfair to sector 1, we have already seen that entry of sector 2 can significantly reduce the level of unemployment. Hence the resulting reduction in welfare taxes can in principle reduce the total taxes for sector 1, and thereby increase their profits.

To formalize these observations, observe first that if sector 1 pays all taxes for both sectors, then for any level of sector 2 employment, L_2, the employee *welfare tax* for sector 1 in (5.1) is now seen to have the new form

$$t(L_1, L_2) = \max\left\{0,\, w_0 \frac{N - L_1 - L_2}{L_1}\right\}. \tag{5.7}$$

Hence for the given market price, $\bar{p}_1$, and sector 2 employment level, $\bar{L}_2$, it follows from (4.29) and (5.3) that an equilibrium employment level, L_1^{**}, for sector 1 must now satisfy the new *two-sector equilibrium condition* (see Fig. 5):

$$\bar{p}_1 F_1'(L_1^{**}) = w_1(L_1^{**}, \bar{L}_2) + t(L_1^{**}, \bar{L}_2) + t_0. \tag{5.8}$$

As in the one-sector case, we again assume that there are at most two solutions to (5.8), so that the unique *profit-maximizing solution*, L_1^{**} (which is always the larger of the two) is the only possible stable equilibrium. Hence we seek conditions under which this profit-maximizing solution exists, and yields an equilibrium for the whole system—which, in addition, offers sector 1 firms some profit incentive for subsidizing the taxes of sector 2. To ensure that $\bar{L}_2$ continues to be the equilibrium employment level for sector 2, we first require that L_1^{**} satisfy (4.7), i.e., that

$$L_1^{**} < N - \bar{L}_2. \tag{5.9}$$

Next, to ensure the appropriate ordering of equilibrium utility levels hypothesized in the job-turnover model, we also require that

$$\underline{w} - e_2 - \tau\left(\frac{1+\alpha}{2}\right)\bar{L}_2 < w_1(L_1^{**}, \bar{L}_2) - e_1 - \tau L_1^{**} \tag{5.10}$$

hold for the given minimum wage, $\underline{w}$. Hence if L_1^{**} satisfies (5.8), (5.9), (5.10), and in addition yields positive profits to all sector 1 firms, then L_1^{**}

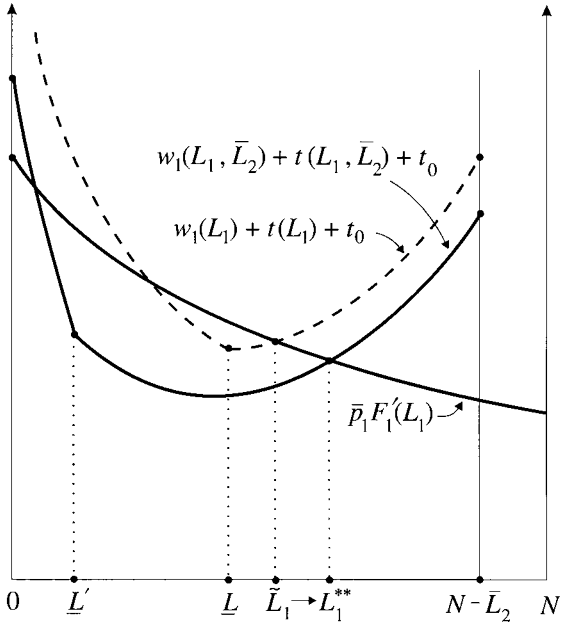

FIG. 5. Two-sector equilibrium with taxation.

will now be designated as a *two-sector equilibrium for the tax case* given $(\bar{p}, \underline{w}, \bar{L}_2)$. This positive-profit condition for sector 1 is of course implied by the stronger condition that sector 1 profits be larger that in the one-sector case (which were positive by hypothesis). This latter requirement is in turn seen from (5.2) to be equivalent to the condition that

$$w_1(L_1^{**}, \bar{L}_2) + t(L_1^{**}, \bar{L}_2) < w_1(\tilde{L}_1) + t(\tilde{L}_1) \tag{5.11}$$

[i.e., that the "total wage level", $w_j + t(\cdot) + t_0$, for sector 1 firms be smaller under L_1^{**} in the two-sector case than under $\tilde{L}_1$ in the one-sector case]. Hence we now say that a two-sector equilibrium, L_1^{**}, *offers profit incentives for sector* 1 if and only if L_1^{**} also satisfies (5.11). Finally, we again say that equilibrium, L_1^{**}, *improves the unemployment situation* iff both total employment and unemployment utility are increased, i.e., iff (i) $L_1^{**} + \bar{L}_2 > \tilde{L}_1$ and (ii) $U_0^{**} > \tilde{U}_0$ $(\equiv \bar{U}_0)$.

With these definitions, our objective is again is to show that if the minimum-wage level, $\underline{w}$, is not too large, then such equilibria are guaranteed. For sake of comparison with the no-tax case, it is convenient to assume that the same minimum wage is admissible for both cases. Hence, if the upper bound in (5.5) is again denoted by $\bar{m}$, and if we now consider the following more stringent upper bound,

$$\tilde{m} = \min\left\{\frac{S(\tilde{L}_1)}{\theta(\tilde{L}_1, \bar{L}_2)}, \frac{w_0 \bar{L}_2}{\tilde{L}_1[1 - \theta(\tilde{L}_1, \bar{L}_2)]}, \bar{m}\right\} > 0, \tag{5.12}$$

then a minimum-wage level, $\underline{w}$, is said to be *admissible under taxation* iff

$$\left[w_0 + e_2 + \tau\left(\frac{1+\alpha}{2}\right)\bar{L}_2\right] < \underline{w} < \left[w_0 + e_2 + \tau\left(\frac{1+\alpha}{2}\right)\bar{L}_2\right] + \tilde{m}. \tag{5.13}$$

As a parallel to Theorem 4.1 above, it can then be shown that (see [34]):

THEOREM 5.1. *For each minimum wage level, $\underline{w}$, admissible under taxation, there exists a unique two-sector equilibrium, L_1^{**}, for the tax case given $(\bar{p}_1, \underline{w}, \bar{L}_2)$. In addition, each such equilibrium offers profit incentives for sector* 1 *and also improves the unemployment situation.*

Finally we note that in this case it can also be shown that $L_1^{**} - \tilde{L}_1 > 0$, and hence that the total employment increase is always *greater* than $\bar{L}_2$. Thus, unlike the no-tax case, entry of the secondary sector now has *positive* indirect employment effects on the primary sector.

6. CONCLUDING REMARKS

In this paper we have shown that demand shocks in the primary sector can indeed give rise to the formation of a secondary sector which absorbs a portion of the unemployment, and thereby softens the effects of the downswing. This result is consistent with the endogenous theory of labor dualism (discussed in the Introduction), and is supported by empirical evidence.[16] In addition, we have shown that when differences in monitoring worker productivity exist between the primary and secondary sectors, both the wages and net incomes of identical workers in each sector may differ. Moreover, so long as some degree of movement between these sectors is possible, such differences will persist in the steady state. This result suggests one possible explanation for the inter-industry wage differentials which are often observed among equally skilled workers (for example in [7]). It also shows that there need be little direct labor competition between these sectors, since workers always prefer to work in the primary sector whenever possible. But if the prevailing minimum-wage level exceeds the welfare payment to nonworkers, then jobs in the secondary sector will still be

[16] For example [29] shows that during the downswing of the U.S. economy during the past fifteen years, most jobs created have been secondary jobs.

preferable to unemployment. Hence this sector will have an indirect effect on primary sector employment—for the presence of a secondary sector will reduce the disciplinary effect of possible job loss in the primary sector, and thereby lead to an increase in the efficiency wage and a corresponding reduction in labor demand by the primary sector.

As mentioned in the Introduction, a major focus of this paper has been on the interaction effects between urban labor markets and spatial structure. On the one hand, the presence of space cost differentials [(4.25), (4.26)] between sectors is shown to influence not only wage differentials but also the level of unemployment itself.[17] In addition, the spatial concentration of unemployment is shown to provide an added incentive for the formation of a secondary-sector. Conversely, secondary-sector formation gives rise in the present model to an SEC which transforms both the residential and commuting structure of the city. In particular, when all primary employment is concentrated in the CBD, the secondary sector is motivated to locate at the outskirts of the city (not at the edge, but just close enough so that the boundary of its commuting shed is at the edge.)[18] Hence, in the presence of a spatially segregated unemployment pool, these secondary-sector jobs tend follow people.[19] The transformed urban landscape is completed by a "buffer zone" of unemployed (if any remain) which separates the two employment zones. But while each of these zones remains fixed, there will continue to be a steady-state flow of individuals moving between zones, as their employment status changes. Such interactions between space structure and labor market structure are even more pronounced when migration between cities is possible. These effects were studied for multiple-city systems with single-sector labor markets in [38], and will be extended to the case of dual labor markets in subsequent work.

Finally, we have shown that when increased unemployment levels are not sufficient to induce the formation of secondary-sector activities, local governments may be strongly motivated to subsidize such activities (as is often observed in practice).[20] With respect to tax subsidies in particular, the present model shows that in cases where welfare payments are locally financed by taxes and where such taxes are the only deterrent to entry by the secondary sector, primary-sector firms may in fact find it profitable to subsidize all secondary-sector taxes themselves, in order to reduce their total tax burden.

[17] This effect on unemployment is illustrated numerically in Example 1 of [38].

[18] This location behavior is consistent with the empirical observation that suburbanization of firms often follows the suburbanization of residents (see for example [35]).

[19] This is in contrast to most multicentric models, such as in [12], where there is no unemployment, and where people always follow jobs.

[20] This is also consistent with the theoretical models of [4] and [3], where local governments compete to attract new firms.

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